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A Comprehensive Review of Transmuted Distributions

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Abstract

In recent years, the development of flexible statistical distributions has gained considerable attention due to the increasing complexity of real-world data. Among various generalization techniques, the transmuted family of distributions has emerged as an effective and mathematically tractable approach for enhancing classical distributions. This family is generated using the quadratic rank transmutation map (QRTM), which introduces an additional parameter to control skewness and kurtosis. The present paper provides a comprehensive review of transmuted distributions, discussing their theoretical foundation, historical development, major contributions, and applications. The study also classifies the existing literature and identifies potential areas for future research.

1. Introduction

The modeling of real-life data often requires distributions that can capture complex characteristics such as skewness, heavy tails, and non-monotonic hazard rate functions. Classical distributions, although widely used, are often inadequate in representing such complexities. As a result, statisticians have developed several methods to generalize existing distributions by introducing additional parameters.

One such approach is the transmutation method, which provides a simple yet powerful mechanism to extend baseline distributions. The concept of transmuted distributions was introduced through the quadratic rank transmutation map (QRTM), which modifies a given cumulative distribution function by incorporating a transmutation parameter. This approach has gained popularity because it retains the simplicity of the original distribution while significantly improving its flexibility.

Over the past decade, a large number of transmuted distributions have been proposed and applied in diverse fields such as reliability engineering, survival analysis, finance, and environmental studies. This paper aims to systematically review these developments and present a unified perspective on the subject.

2. The Transmuted Family of Distributions

Let $G(x)$ denote the cumulative distribution function (cdf) of a baseline distribution. The transmuted distribution is defined by applying the quadratic rank transmutation map as follows:

$$F(x) = (1+\lambda)G(x) - \lambda G(x)^2, \quad |\lambda| \le 1$$

Here, □ is the transmutation parameter that governs the deviation from the baseline model. When □ □ 0, the transmuted distribution reduces to the original distribution, indicating that the baseline model is a special case of the transmuted family.

The corresponding probability density function (pdf) can be obtained by differentiating the above expression. The introduction of the parameter □ allows for greater flexibility in modeling data, particularly in controlling skewness and tail behavior. Moreover, the transmuted distributions preserve many desirable properties of the parent distributions, making them analytically convenient.

3. Historical Development

The origin of transmuted distributions can be traced back to the work of Shaw and Buckley (2007), who introduced the quadratic rank transmutation map as a tool for generating new distributions from existing ones. Their work laid the foundation for subsequent developments in this area.

Following this, Aryal and Tsokos (2011) made a significant contribution by proposing the transmuted Weibull distribution, which demonstrated improved performance in modeling lifetime data. This work sparked considerable interest among researchers, leading to the development of numerous transmuted models based on different baseline distributions.

Between 2013 and 2017, the literature on transmuted distributions expanded rapidly. Researchers proposed a variety of models, including transmuted versions of the Rayleigh, Lomax, Pareto, and inverse Weibull distributions. During this period, emphasis was placed on deriving statistical properties and demonstrating the superiority of these models through real data applications.

In recent years, the focus has shifted towards developing more generalized and hybrid models. These include combinations of transmutation with other generator families such as the Marshall– Olkin and exponentiated families. Such developments have further enhanced the flexibility and applicability of transmuted distributions.

4. Review of Literature

A substantial body of literature has emerged on transmuted distributions over the past decade. Shaw and Buckley (2007) introduced the fundamental concept of transmutation, which was later extended by Aryal and Tsokos (2011) through the transmuted Weibull distribution. Their work demonstrated the practical utility of the transmutation approach in reliability analysis.

Khan and King (2013) proposed the transmuted modified Weibull distribution and studied its statistical properties in detail. Around the same time, Merovci (2013) introduced the transmuted Rayleigh distribution and highlighted its applications in engineering data. Elbatal (2013) further extended this line of work by developing the transmuted inverse Weibull distribution.

Ashour and Eltehiwy (2013) contributed by proposing the transmuted Lomax distribution, which has been widely used in modeling heavy-tailed data. In subsequent years, several researchers focused on extending transmuted distributions by combining them with other generator techniques. For instance, Afify et al. developed transmuted Marshall–Olkin families, which provide additional flexibility in modeling complex datasets.
Oguntunde and Adejumo (2015) studied the transmuted exponential distribution and demonstrated its superiority over classical models in fitting real data. Nofal et al. and Yousof et al. made further contributions by introducing exponentiated and generalized transmuted families.

More recently, researchers have explored weighted and compounded versions of transmuted distributions. These models have been applied to environmental and financial data, showing improved performance compared to traditional distributions.

5. Classification of Transmuted Distributions

The existing literature on transmuted distributions can be broadly classified based on the choice of baseline distribution, the type of generator used, and the nature of the data being modeled.

Based on the baseline distribution, transmuted models include transmuted exponential, Weibull, Rayleigh, Lomax, and gamma distributions. Each of these models inherits the basic structure of the parent distribution while gaining additional flexibility through the transmutation parameter.

From the perspective of generator techniques, transmuted distributions have been combined with other families such as exponentiated, beta-generated, and Marshall–Olkin families. These hybrid models often exhibit superior flexibility and can capture a wider range of data behaviors.

In terms of data type, both continuous and discrete transmuted distributions have been developed. While most studies focus on continuous models, there has been growing interest in discrete transmuted distributions for count data analysis.

6. Statistical Properties

A significant portion of the literature on transmuted distributions is devoted to the study of their statistical properties. Researchers have derived expressions for moments, moment generating functions, and quantile functions. These properties are essential for understanding the behavior of the distributions and for facilitating parameter estimation.

The hazard rate function is another important aspect that has been extensively studied. Transmuted distributions are capable of exhibiting various shapes of hazard functions, including increasing, decreasing, and bathtub-shaped forms. This makes them particularly useful in reliability and survival analysis.

In addition, properties such as entropy measures, order statistics, and stochastic ordering have been investigated. These studies provide deeper insights into the theoretical characteristics of transmuted distributions.

7. Applications

Transmuted distributions have been successfully applied in a wide range of fields. In reliability engineering, they are used to model failure times and to analyze systems with complex hazard rate structures. Their flexibility makes them suitable for modeling components with varying failure mechanisms.

In survival analysis, transmuted distributions are employed to analyze lifetime data, particularly in medical and biological studies. They provide better fits to data with skewed or heavy-tailed characteristics.

Environmental studies also benefit from transmuted models, especially in the analysis of rainfall, temperature, and pollution data. In finance and insurance, these distributions are used to model risk and to analyze claim sizes.

Overall, the ability of transmuted distributions to provide improved goodness-of-fit has made them a valuable tool in applied statistics.

8. Limitations and Research Gaps

Despite the extensive development of transmuted distributions, certain limitations remain. One of the major gaps in the literature is the lack of work on multivariate transmuted distributions. Most existing studies focus on univariate cases, limiting their applicability in multivariate settings.

Another area that requires attention is parameter estimation. While maximum likelihood estimation is commonly used, there is a need for more robust and efficient estimation techniques, particularly for small sample sizes.

Furthermore, the application of transmuted distributions in statistical process control is still limited. Given their flexibility, these distributions have significant potential in developing control charts and monitoring industrial processes.

9. Future Research Directions

Future research on transmuted distributions can focus on several promising areas. The development of multivariate and regression-based transmuted models would greatly enhance their applicability. Additionally, integrating transmuted distributions with Bayesian methods and machine learning techniques could open new avenues for research.

Another important direction is the application of transmuted distributions in statistical process control, particularly in designing control charts for non-normal data. This area is highly relevant and offers significant potential for further exploration.

10. Conclusion

The transmuted family of distributions represents a significant advancement in statistical modeling. By introducing a simple yet effective transformation, these distributions provide enhanced flexibility while retaining mathematical tractability. Over the years, a large number of transmuted models have been developed, and their applications have expanded across various fields.

Despite the progress made, there are still several opportunities for further research. Addressing the existing gaps and exploring new applications will ensure the continued growth and relevance of transmuted distributions in statistical science.

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