

Airline Crew Scheduling Using Set Partitioning and Column Generation: A Report on Barnhart et al. (2003) with an Indian Context

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Abstract—

Airline crew pay depends on rules that limit who may fly which flight, and a weak crew plan is costly to fix once flights are running. The regulator's inquiry into the cancellation of 2,507 IndiGo flights in three days of December 2025 pointed to over-optimised rosters with thin buffers. This report studies the chapter "Airline Crew Scheduling" by Barnhart, Cohn, Johnson, Klabjan, Nemhauser and Vance (2003) and sets it out as an operations research case study. The chapter models crew pairing as a set partitioning problem and solves it by column generation and branch-and-price. It then extends the models to crew assignment, recovery from disruption and robust planning. We re-work the chapter's seven-flight example and check it by enumeration and a linear programming solver. Only one set of pairings covers every flight exactly once, at 9 hours of pay for 7 hours of flying. The linear programming bound is 8 without the crewbase constraints and 9 with them. We also tabulate the results quoted in the chapter, such as a fall in the gap to the flying-time bound from 10 to 15 per cent to 1 to 2 per cent, and relate them to the Indian case. The chapter is strongest on method and weakest on open data and on rules outside the United States.

Keywords— airline crew scheduling; crew pairing; set partitioning; column generation; branch-and-price; robust scheduling; flight duty time limitations

I. INTRODUCTION AND PROBLEM TYPE

A. Background

An airline cannot fly a flight without a legal crew. Cockpit crews are qualified for one fleet type or a small family of types, and they are not salaried. They are paid for the time they fly, plus extra for long waits on the ground and for rest away from home (Barnhart et al., 2003). The total flying time of the schedule is fixed, so it acts as a lower bound on crew cost, and the aim of planning is to reduce the payment made above it, which the chapter calls pay-and-credit. The stakes are high. According to the chapter, solutions to the daily crew pairing problem were typically 10 to 15 per cent above the flying-time bound about a decade before 2003 and are now within 1 to 2 per cent of it, a gain it puts at about \$50 million a year for a large airline.

Crew planning comes last in the airline planning sequence. Schedule design, fleet assignment and maintenance routing are solved first, and crew scheduling then splits into a pairing problem and an assignment problem (Figure 1). The problem has recently been in the Indian news. The Directorate General of Civil Aviation (DGCA) introduced revised

flight duty time limitations in two phases, from 1 July and 1 November 2025, raising weekly rest to 48 hours and limiting night landings to two, against six earlier (Press Trust of India [PTI], 2025; Storyboard18, 2025). Between 3 and 5 December 2025, IndiGo cancelled 2,507 flights and delayed 1,852, and more than three lakh passengers were stranded (Press Information Bureau [PIB], 2026).

B. Problem Statement

An airline has to cover every flight of a fleet type with a legal crew at the lowest possible pay. Flights are first strung into duty periods, which are a day's work, and duties are strung into pairings, which are trips that start and end at the crew's home base. The number of legal pairings is huge. A domestic problem with several hundred flights on a hub-and-spoke network has billions of them (Barnhart et al., 2003). The planner must pick the set of pairings that covers each flight exactly once at the least cost, then give the pairings to individuals as monthly schedules, and repair the plan within minutes when flights are delayed or cancelled.

(a) Airline planning sequence



(b) Building blocks of a crew schedule

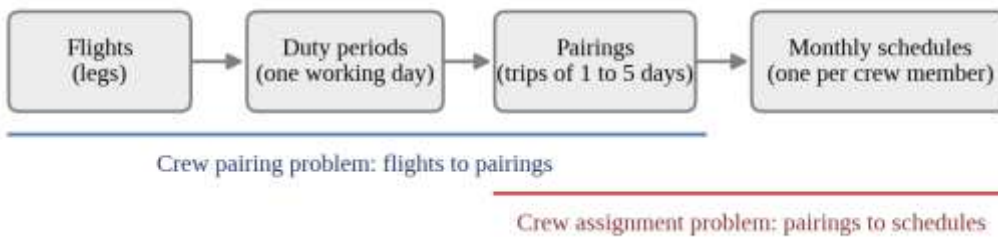


Figure 1. Airline planning sequence and the building blocks of a crew schedule (after Barnhart et al., 2003).

Crew decision	OR model	Solution method	Used in this report
Which pairings cover every flight exactly once at least cost?	Set partitioning (integer programming)	Column generation and branch-and-price	Sections IV-B and V
Is the LP bound close to the integer optimum?	LP relaxation of the same model	LP solver and follow-on branching	Sections V-B and V-C
Which monthly schedule should each crew member fly?	Crew rostering (set covering type)	Column generation, constraint programming for pricing	Section IV-C (described only)
How is the plan repaired after a disruption?	Crew recovery model with cancellation and deadhead variables	Restricted model for the disrupted crews	Section IV-C (described only)
Will the plan still be cheap when it is flown?	Robust pairing (expected cost, move-up crews)	Simulation, Lagrangian decomposition	Section VI-C

Table I. Operations Research techniques in the crew scheduling chapter.

C. Objectives

- To explain the airline crew scheduling problem and its place in airline planning, using the chapter as the main source.
- To express crew pairing as a set partitioning model and to re-work the chapter's seven-flight example.
- To describe column generation and branching, and to check the example by enumeration, linear programming and branching.
- To tabulate the results quoted in the chapter, relate them to the Indian case of 2025, and assess the chapter critically.

D. Scope

This report covers one chapter of the Handbook of Transportation Science, which is about 49 pages in the version supplied to us. It follows the chapter in treating cockpit crews. The numerical work is limited to the chapter's own seven-flight example, and no new airline data were collected. Column generation, the rostering model and the recovery model are described and discussed but not implemented. The Indian case is used only as context.

II. REAL-LIFE CONTEXT: AIRLINE CREW SCHEDULING IN PRACTICE

A. Industry Data

Table II gives the published figures used to set the scale of the problem. The first six rows are reported in the chapter. The last two come from the Indian regulator and the press.

Item	Period	Figure	Source
Size of a pairing problem	Early 2000s	Several hundred flights on a U.S. hub-and-spoke network give billions of feasible pairings	Barnhart et al. (2003)
First large LP solved by enumeration	1992	An 800-flight daily problem, with 5.5 million pairings enumerated	Anbil et al. (1992), via Barnhart et al. (2003)
Gap to the flying-time bound	Early 1990s and 2003	10 to 15 per cent then, at most 1 to 2 per cent in 2003; about \$50 million a year for a large airline	Barnhart et al. (2003)
Pay-and-credit, planned and operated	Early 2000s	Large fleets: below 1 per cent planned, about 4 per cent operated. Smaller fleets: about 3 per cent planned, about 8 per cent operated	Barnhart et al. (2003)
Delays in the United States	1995 to 2000	Delay minutes up 11 per cent from 1995 to 1999. In summer 2000 the best airline had 25 per cent of flights delayed by 15 minutes or more	Bond (2000), cited in Barnhart et al. (2003)
Solution time	Early 2000s	300 legs (hub-and-spoke) or 2,000 legs (point-to-point) took 10 to 20 hours of CPU time to solve even approximately	Barnhart et al. (2003)
DGCA flight duty time limitations	July to Nov 2025	Phased from 1 July, complete on 1 November 2025. Weekly rest of 48 hours; night landings limited to two, against six earlier	PTI (2025); Storyboard18 (2025)

Item	Period	Figure	Source
IndiGo disruption	3 to 5 Dec 2025	2,507 flights cancelled and 1,852 delayed; over three lakh passengers stranded; penalty of Rs 22.20 crore and a Rs 50 crore bank guarantee	PIB (2026)

Table II. Published figures on crew scheduling and its costs.

The chapter's own numbers show why plans suffer in operation. For a large fleet, pay-and-credit rises from below 1 per cent of flying time in the plan to about 4 per cent when flown, which is at least four times the planned excess. For smaller fleets it rises from about 3 to about 8 per cent, or about two and a half times.

B. Pay Rules and Pay-and-Credit

The chapter describes how U.S. crews are paid. A duty period is a day's work, and its pay is the largest of three quantities: the flying time, a fraction of the elapsed time of the duty, and a minimum guarantee (Barnhart et al., 2003, Section 14.2.1).

$$b_d = \max \{ f_d \times \text{elapse}, \text{fly}, \text{min}_{\text{guar}} \}$$

Here b_d is the pay of duty d in minutes, f_d is the fraction (the chapter gives $5/8$ as an example), elapse is the elapsed time of the duty, fly is its flying time and min_{guar} is the minimum guarantee. The cost of a pairing p is the largest of a fraction of its time away from base (TAFB), a minimum guarantee per duty, and the sum of its duty pays, plus the cost of meals and lodging for the rest between duties:

$$c_p = \max \{ f_p \times \text{TAFB}, n_{dp} \times \text{mg}, \sum_{d \in p} b_d \} + \sum e(d, d')$$

where n_{dp} is the number of duties in p , mg is the guarantee per duty and $e(d, d')$ is the extra cost of the rest between consecutive duties d and d' . Pay-and-credit is the amount by which these costs exceed the flying time. Pairings must also obey rules on rest, duty length and time away from base, such as the 8-in-24 rule of the U.S. regulator, which requires extra rest if a pairing has more than 8 hours of flying in any 24 hours (Barnhart et al., 2003).

To see how pay-and-credit arises, take three duty periods of the chapter's example (Section IV-A) and the $5/8$ fraction. We assume the minimum guarantee is no more than two hours, so that it does not bind. Table III shows that a duty with long waits is paid more than it flies.

Duty	Flights	Flying (min)	Elapsed (min)	5/8 of elapsed (min)	Duty pay (min)	Pay-and-credit (min)
D8	1, 2	120	180	112.5	120	0
D10	2, 3	120	240	150	150	30
D9	1, 7, 3	180	360	225	225	45

Table III. Duty pay under the $5/8$ rule (assumed minimum guarantee does not bind).

Duty D9 flies three hours but is spread over six, so it is paid 3 hours 45 minutes. This is why planners prefer short sits. The table also shows that the costs in the chapter's example cannot have come from the $5/8$ rule. Pairing P2 contains D9 and flight 5, which would cost at least 4 hours 45 minutes, while the chapter uses 4. The example costs are therefore best read as given inputs.

C. Why Operations Research Is Suitable

- The decision is a matter of numbers: which pairings to fly, and how much work to give each crewbase.
- Costs can be measured in minutes of pay from flight times and pay rules.
- Rules on rest, duty length and night work shrink the set of legal options so much that good plans cannot be found by hand, and an error is a safety and legal risk.

- The model has to be solved again whenever the flight schedule changes or a regulator changes the rules, as the DGCA did in 2025.

III. LITERATURE REVIEW

A. Source and Counting of the Literature

This report studies a single chapter, so its literature is the chapter's own bibliography. We counted it from the reference list of the version supplied to us. It has 69 references. Figure 2 shows them by year of publication. Fifty-two of them, or 75 per cent, appeared in 1995 or later, and the count peaks at 12 in 1998, so the chapter reflects the burst of work in the late 1990s. Six appeared before 1990, the oldest being Ryan and Foster (1981). Nothing is cited after 2002, and some items are marked as "to appear", so the literature stops at the date of writing.

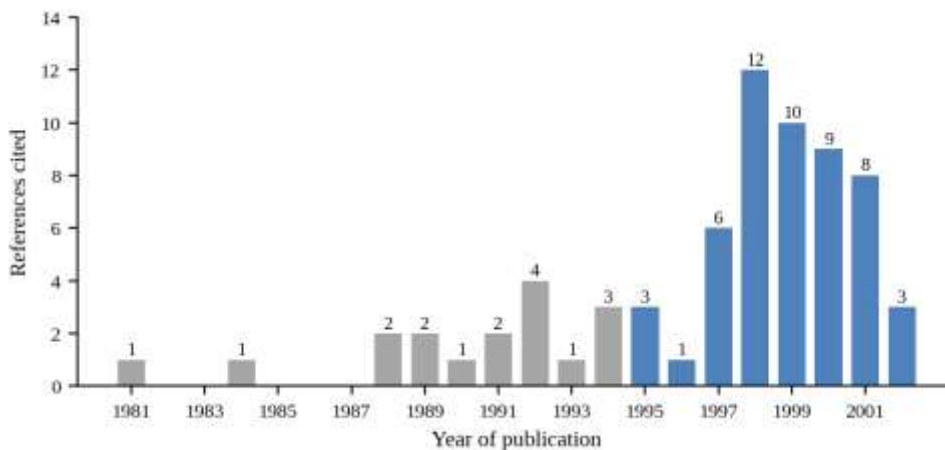


Figure 2. References cited in the chapter, by year of publication (n = 69). Bars from 1995 onwards are shaded blue.

The count has limits. It rests on the authors' own choice of sources. Of the 69 references, 20 (29 per cent) have at least one of the chapter's six authors among their authors, and 26 (38 per cent) were technical reports, conference papers, presentations, a thesis or articles marked "to appear" at the time. Several of these were published later (for example Cohn & Barnhart, 2003), so the second figure says more about the date of the chapter than about the quality of the work.

B. From Local Search to Set Partitioning

Until the 1990s the state of the art was local improvement. Starting from a feasible plan, a heuristic picked a few pairings, listed every pairing for their flights and solved the small set partitioning problem exactly (Gershkoff, 1989; Anbil et al., 1991). This worked, but it looked at a small part of the network at a time and gave no lower bound (Barnhart et al., 2003). Hoffman and Padberg (1993) listed a subset of pairings up front and solved the integer program over it.

C. Solving the LP Relaxation

Anbil et al. (1992) solved the LP over 5.5 million pairings of an 800-flight problem with an approach called SPRINT, which loads a few thousand columns at a time. Bixby et al. (1992) combined interior point and simplex methods on a very large model. Column generation then replaced listing (Anbil et al., 1994; Desaulniers et al., 1998). Work on the master problem dealt with degeneracy. Du Merle et al. (1999) added penalised slack variables to stabilise the dual values, and Barahona and Anbil (1999) proposed the volume algorithm, which they report is faster than interior point and simplex methods.

D. Pricing and Network Models

Lavoie et al. (1988) and Desrochers and Soumis (1988) price columns with multi-label shortest path methods on networks of duties or flights. Anbil et al. (1994) used a duty network for international problems with two to three times as many duties as flights. Software vendors often prefer to list pairings instead, because a change in a rule can force major changes to shortest path code (Barnhart et al., 2003). Makri and Klabjan (2001) select columns by the cost of a pairing divided by the sum of its dual values, a score introduced by Bixby et al. (1992).

E. Integer Solutions and Branching

Ryan and Foster (1981) gave a branching rule for set partitioning in which two rows are covered by the same column on one branch and by different columns on the other. For crew pairing the chapter modifies it into branching on follow-ons, which forces two flights to be consecutive or not. Klabjan et al. (2001b) proposed timeline branching and random pairing generation. Vance et al. (1997a) and Barnhart et al. (1998) developed branch-and-price, which generates columns throughout the search tree. Marsten (1994) fixes variables with values close to one, and Andersson et al. (1998) solve the integer program with the Lagrangian method of Wedelin (1995).

F. Rostering, Integration, Recovery and Robustness

For rostering, Ryan (1992) solved problems with 55 crew members and 120 pairings, and Gamache et al. (1998) describe the preferential bidding system of Air Canada. Junker et al. (1999) used constraint programming to price columns when the rules were too complex for shortest path methods. Crew pairing has been linked with maintenance routing (Cordeau et al., 2001; Cohn & Barnhart, 2003) and with flight retiming (Klabjan et al., 2002). Recovery has been studied by Teodorovic and Stojkovic (1990), Wei and Yu (1997), Song et al. (1998), Stojković et al. (1998) and Lettovský et al. (2000), and the chapter notes that little has been published. For robustness, Schaefer et al. (2000) use expected pairing costs found by simulation, Yen and Birge (2000) and Ehrgott and Ryan (2001) reward slack in connections, and Klabjan et al. (2001c) maximise the number of move-up crews.

G. Later Developments

Kasirzadeh et al. (2017) review the problem again, formulate personalised cockpit crew scheduling as a set covering problem solved by column generation, and describe data sets from a major U.S. carrier that are available on the internet, to give later researchers a common basis. This is the kind of open benchmark that the chapter lacks.

H. Summary of the Studies

Study	Focus	Relevance to this report
Gershkoff (1989); Anbil et al. (1991)	Local improvement heuristics	The starting point the chapter moves beyond
Anbil et al. (1992); Bixby et al. (1992)	Solving the LP of very large pairing models	Shows the size of the problem
Lavoie et al. (1988); Desrochers and Soumis (1988); Makri and Klabjan (2001)	Pricing by shortest path or by listing	Basis of Section V-D
Ryan and Foster (1981); Klabjan et al. (2001b); Vance et al. (1997a)	Branching and branch-and-price	Basis of Sections V-C and V-E
Ryan (1992); Gamache et al. (1998); Junker et al. (1999)	Crew rostering and constraint programming	Assignment model, Section IV-C
Cordeau et al. (2001); Cohn and Barnhart (2003); Klabjan et al. (2002)	Integration with maintenance and schedule	Section VI-C
Lettovský et al. (2000)	Crew recovery model	Section IV-C
Schaefer et al. (2000); Klabjan et al. (2001c)	Robust crew pairing	Sections VI-C to VI-E
Kasirzadeh et al. (2017)	Later survey and public data sets	Sections III-G and VII

Table IV. Summary of the literature reviewed.

I. Research Gap

The chapter and the studies it covers leave three gaps, and this report addresses them in a limited way. The first is geographic. The pay rules, the 8-in-24 rule and the hub-and-spoke emphasis are those of the United States, and none of the 69 references has India in its title. The 2025 rule change and the IndiGo disruption show that Indian rules and Indian operations deserve the same analysis.

The second gap is evidence. The chapter gives its main results as ranges, such as 10 to 15 per cent, 1 to 2 per cent and 10 to 40 per cent, without instance data, and its worked example uses stylised costs. A reader cannot reproduce the numbers from the chapter alone. The third gap is depth on recovery and robustness. The authors themselves describe their coverage of these topics as a beginning, and the link between a change in regulation and the crew plan is not discussed.

This report responds in three ways. It re-works the chapter's example step by step and checks each figure. It tabulates the reported results with their sources and conditions. And it sets the chapter's models beside the DGCA rule change and the IndiGo disruption of 2025.

IV. MATHEMATICAL MODELLING

A. Flights, Duty Periods and Pairings

The chapter illustrates the pairing model with seven flights between four airports, A, B, C and D, all in one time zone. Airports A, C and D are crewbases. Table V gives the flights. Each flight lasts one hour, and costs are read as hours of pay.

Flight	From	To	Departs	Arrives	Operates
1	A	B	08:00	09:00	Every day except Saturday and Sunday
2	B	C	10:00	11:00	Every day
3	C	D	13:00	14:00	Every day except Sunday
4	C	A	15:00	16:00	Every day
5	D	A	15:00	16:00	Every day except Saturday
6	A	B	17:00	18:00	Every day
7	B	C	11:00	12:00	Every day except Saturday and Sunday

Table V. Flights of the chapter's example.

Ten duty periods are valid: D1 to D7 are the seven single flights, $D8 = \{1, 2\}$, $D9 = \{1, 7, 3\}$ and $D10 = \{2, 3\}$. Six pairings can be built from them, shown in Table VI. Pairing P6 covers flight 7 twice and is dropped from the daily problem. A further pairing $\{D4, D1, D2\}$ is left out because it covers the same flights as P1 and the chapter assumes it costs more (Barnhart et al., 2003).

Pairing	Duty periods	Flights covered	Crewbase	Flying hours	Cost (hours of pay)
P1	D4, D8	4, 1, 2	C	3	4
P2	D9, D5	1, 7, 3, 5	A	4	4
P3	D5, D6, D10	5, 6, 2, 3	D	4	4
P4	D4, D6, D7	4, 6, 7	C	3	4
P5	D1, D7, D4	1, 7, 4	A	3	5
P6	D5, D7, D9	5, 7, 1, 7, 3	-	-	Not used (flight 7 twice)

Table VI. Pairings of the daily problem and their costs.

B. The Set Partitioning Model

Let F be the set of flights, P the set of feasible pairings, c_p the cost of pairing p , and $y_p = 1$ if p is chosen and 0 otherwise. The crew pairing model of the chapter is

$$\text{Minimise } Z = \sum_{p \in P} c_p y_p$$

subject to $\sum_{p: i \in p} y_p = 1$ for every flight $i \in F$, and $y_p \in \{0, 1\}$

Decision variables: $y_1, \dots, y_5 \in \{0, 1\}$, where $y_j = 1$ means that pairing P_j is flown.

Objective function: minimise the total pay

$$\text{Minimise } Z = 4y_1 + 4y_2 + 4y_3 + 4y_4 + 5y_5$$

Cover constraints (one per flight):

$$\text{Flight 1: } y_1 + y_2 + y_5 = 1$$

$$\text{Flight 2: } y_1 + y_3 = 1$$

$$\text{Flight 3: } y_2 + y_3 = 1$$

$$\text{Flight 4: } y_1 + y_4 + y_5 = 1$$

$$\text{Flight 5: } y_2 + y_3 = 1$$

$$\text{Flight 6: } y_3 + y_4 = 1$$

$$\text{Flight 7: } y_2 + y_4 + y_5 = 1$$

Crewbase balance constraints: the chapter asks for between 3 and 6 hours of work at crewbases A and D and at most 5 hours at C. The coefficients are the flying hours of the pairings based at each crewbase.

$$\text{Crewbase A: } 3 \leq 4y_2 + 3y_5 \leq 6$$

$$\text{Crewbase C: } 0 \leq 3y_1 + 3y_4 \leq 5$$

$$\text{Crewbase D: } 3 \leq 4y_3 \leq 6$$

Assumptions:

- Every flight operates every day (the daily problem), so each pairing can be repeated daily.
- All pairings in the model are legal and their costs are given. Generating pairings is not part of the example.
- Each flight is flown by exactly one crew, and the model has no deadhead variable.
- Only crews of one fleet type are considered.

C. Crew Assignment and Recovery Models

The crew assignment (rostering) model of Gamache and Soumis, as given in the chapter, chooses one schedule for each crew member so that every dated pairing p has at least n_p crew. Let K be the crew members, S^k the schedules feasible for member k , $\gamma_p^s = 1$ if schedule s contains pairing p , and $x_s^k = 1$ if member k receives schedule s :

Minimise $\sum_{k \in K} \sum_{s \in S^k} c_s^k x_s^k$

$\sum_k \sum_s \gamma_p^s x_s^k \geq n_p$ for all p ; $\sum_{s \in S^k} x_s^k = 1$ for all k ; $x_s^k \in \{0, 1\}$

The recovery model of Lettovský et al. (2000) has the same structure, but each pairing is specific to a crew and starts when that crew next becomes available. It adds variables for cancelling a flight (y_1), for crews deadheading on a flight (w_1) and for a crew left without a pairing and returned to base (v_k). The objective is the total of the cost of adjusted pairings, cancellation costs, deadhead costs and the cost of returning crews. Each flight must be covered or cancelled, and each crew is either given a pairing or sent home.

D. Note on the Data

The flights, duty periods, pairings, costs and crewbase limits in Tables V and VI are those printed in the chapter. The calculations in Sections V and VI (enumeration, LP bounds, branching and the weekly table) are ours and were checked in Python. The duty pay illustration in Table III uses the 5/8 fraction that the chapter gives as an example and an assumed minimum guarantee small enough not to bind. The Indian figures come from the regulator's press release and press reports. Most of the chapter's other figures are quoted from the original studies, which we have not been able to check.

V. SOLUTION METHODOLOGY

A. Solving the Daily Problem

With five pairings there are only $2^5 = 32$ combinations, so the example can be solved by checking each one. It is more useful to reason on the constraints, as an integer programming solver does. Table VII sets out the steps.

Step	Reasoning	Result
1	Flights 3 and 5 give the same constraint, $y_2 + y_3 = 1$	Exactly one of P2 and P3 is chosen
2	Try $y_3 = 0$. Then $y_2 = 1$ (flight 3), $y_1 = 1$ (flight 2) and $y_4 = 1$ (flight 6)	Flight 1 would be covered by $y_1 + y_2 = 2$ pairings, so this is infeasible and $y_3 = 1$
3	With $y_3 = 1$, flights 2, 3 and 6 force $y_1 = y_2 = y_4 = 0$	Only P5 is left to choose
4	Flight 1 requires $y_5 = 1$. Flights 4 and 7 are then covered by P5	All seven flights are covered exactly once
5	Cost is $4 + 5 = 9$. Hours of work are 3 at A, 0 at C and 4 at D	Within the limits of 3 to 6, 0 to 5 and 3 to 6

Table VII. Steps in solving the daily problem.

The only feasible partition is therefore $\{P3, P5\}$, which is the optimum quoted in the chapter. Table VIII shows the trips day by day. On any day one crew starts each pairing, and the crews that started on earlier days cover the later duties of the same trip.

Pairing	Day 1	Day 2	Day 3
P3 (base D)	Flight 5: D to A	Flight 6: A to B	Flights 2, 3: B to C to D
P5 (base A)	Flight 1: A to B	Flight 7: B to C	Flight 4: C to A

Table VIII. The optimal pairings, by day of the trip.

$$Z = 4(1) + 5(1) = 9 \text{ hours of pay for 7 hours of flying}$$

B. The LP Relaxation

A solver that ignores integrality gives a bound. Put $y_3 = t$. Flights 3 and 5 give $y_2 = 1 - t$, flight 2 gives $y_1 = 1 - t$, flight 6 gives $y_4 = 1 - t$, and flight 1 then gives $y_5 = 2t - 1$, which also satisfies flights 4 and 7. Since $y_5 \geq 0$, t lies between $1/2$ and 1 , and the cost is

$$Z = 12(1 - t) + 4t + 5(2t - 1) = 7 + 2t$$

The lowest cost is 8, at $t = 1/2$, where $y_1 = y_2 = y_3 = y_4 = 1/2$ and $y_5 = 0$. Without the crewbase constraints the LP bound is therefore 8, one hour below the integer optimum of 9, so the model is not integral by itself. The crewbase constraints change this. Crewbase A needs $4y_2 + 3y_5 = 1 + 2t \geq 3$, which forces $t = 1$, and the LP solution becomes the integer solution. Table IX shows three points on the line.

t	y_1, y_2, y_3, y_4, y_5	Cost	Hours at A	Hours at C	Hours at D	Meets crewbase limits?
0.50	0.5, 0.5, 0.5, 0.5, 0	8.0	2.0	3.0	2.0	No (A and D below 3)
0.75	0.25, 0.25, 0.75, 0.25, 0.5	8.5	2.5	1.5	3.0	No (A below 3)
1.00	0, 0, 1, 0, 1	9.0	3.0	0.0	4.0	Yes

Table IX. LP solutions along $y_3 = t$.

The lesson is general. In this small example the side constraints happen to tighten the LP, but in real problems a fractional solution is common and branching is needed.

C. Branching on Follow-ons

Branching on follow-ons forces two flights to appear consecutively in a pairing, or not (Barnhart et al., 2003, Section 14.4.4.1). Flights 2 and 3 are consecutive only in P3. At the root without the crewbase constraints, where the LP gives the fractional solution above, the two branches are shown in Table X.

Branch	Columns left in the master	Result
Flight 3 must follow flight 2	P3, P4, P5 (P1 and P2 removed)	Partition {P3, P5} with cost 9, an integer solution
Flight 3 must not follow flight 2	P1, P2, P4, P5 (P3 removed)	No feasible partition, so the branch is pruned

Table X. The two branches on the follow-on (flight 2, flight 3).

The first branch gives an integer solution at 9 and the second is infeasible, so the search ends. In a real branch-and-price method new columns would also be generated at each node, and the pricing network would be changed by deleting arcs so that it respects the branching decision.

D. Column Generation and Pricing

For real problems the columns cannot be listed. The chapter's column generation algorithm has three steps (Figure 3):

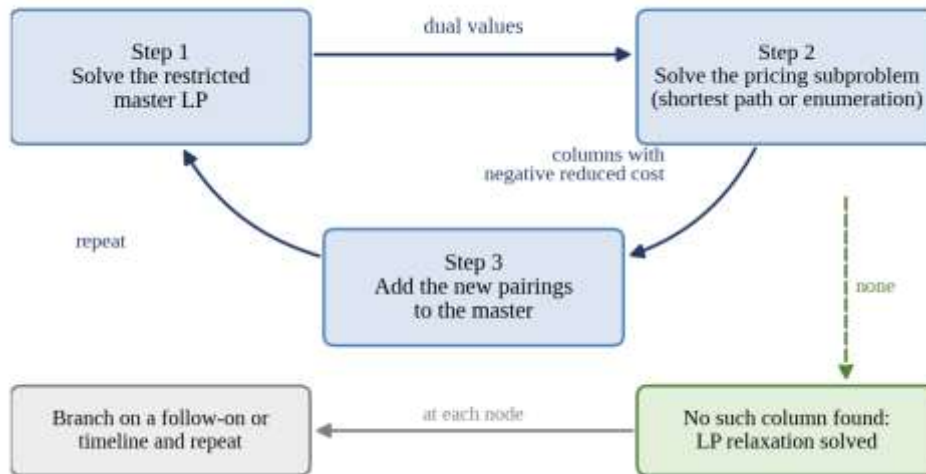


Figure 3. Column generation for the crew pairing LP (after Barnhart et al., 2003).

- **Step 1:** solve the restricted master problem, which holds only some of the pairings, and read its dual values.
- **Step 2:** solve the pricing subproblem to find pairings that may improve the solution. If there are none, stop: the LP is solved.
- **Step 3:** add the new pairings to the master problem and return to Step 1.

Pricing is done on a network. In a flight network each flight is an arc and connection arcs join legal pairs of flights. In a duty period network each possible duty is an arc. A source and a sink are linked to the crewbase, so every legal pairing is a path. The reverse is not true, so rules that the network cannot hold, such as the limit on flying hours in a duty, are tracked with labels. Figure 14.4 of the chapter gives a small example. Each path carries four labels: the flying time of the current duty, its elapsed time, its number of flight segments, and the number of duties in the pairing. Table XI repeats the update.

Path to the departure node of flight C	Flying (h)	Elapsed (h)	Segments	Duties	Kept?
Arrival of flight A	3.0	6.0	2	1	-
Arrival of flight B	4.0	5.0	4	1	-
Via A, connection of 2 hours	3.0	8.0	2	1	Yes
Via B, connection of 1 hour	4.0	6.0	4	1	Yes
A third path (illustration only)	4.0	7.0	4	1	No, dominated

Table XI. Label update and dominance (after Figure 14.4 of the chapter).

Neither of the two real paths dominates the other. One has less flying time and the other has less elapsed time, so both must be kept. A path is dropped only when another path to the same node is at least as good on every label, as the illustrative third path is. The chapter's text gives the labels of the second path as (4.0, 7.0, 4, 1), but 5.0 plus a one-hour connection is 6.0, which is also what its figure shows. The conclusion does not change.

E. Strategies for Integer Solutions

The chapter groups the methods for finding integer solutions into three classes (Barnhart et al., 2003, Section 14.4.4):

- **Columns listed off-line:** a subset of pairings is listed up front and the integer program is solved over it (Hoffman & Padberg, 1993). This is fast, but even moderate problems have billions of variables, so the subset must be very small.

- **LP by column generation, then branch-and-bound on those columns:** examples are Anbil et al. (1994), Ryan (1992) and Klabjan et al. (2001b). There is no guarantee that a good, or even a feasible, solution lies among these columns.
- **Branch-and-price:** columns are generated at every node (Vance et al., 1997a; Barnhart et al., 1998). It needs a branching rule that fits the pricing network, which is why follow-on and timeline branching were developed.

Pairing generation is the most time-consuming step, so parallel methods share out the starting legs among processors (Goumopoulos et al., 1997; Klabjan & Schwan, 2000).

F. Repairing the Weekly Problem

The daily solution assumes every flight runs every day. Flights 1, 3, 5 and 7 do not, so some copies of P3 and P5 are broken. P3 (flight 5 on day 1, flight 6 on day 2, flights 2 and 3 on day 3) is broken when it starts on a Friday or Saturday. P5 (flights 1, 7 and 4 on days 1 to 3) is broken when it starts on a Friday, Saturday or Sunday. The flights left uncovered, which the weekly exceptions problem must cover, are listed in Table XII.

Day	Flights left by broken copies of P3	Flights left by broken copies of P5
Friday	5	1
Saturday	6	-
Sunday	6, 2	4
Monday	2, 3	7, 4
Tuesday	-	4

Table XII. Flights left uncovered by broken pairings in the weekly problem.

These 11 flights match the chapter's table of weekly exceptions. As a check, the weekly schedule has 43 flight segments, and the unbroken copies of P3 (5 start days $\times$ 4 flights = 20) and P5 (4 start days $\times$ 3 flights = 12) cover 32 of them, leaving 11.

G. Verification Using a Solver

To confirm the hand calculation, the model was checked in Python. All 32 combinations of the five variables were tested, and the LP relaxation was solved with and without the crewbase constraints using the HiGHS solver in SciPy. The check found one feasible partition, {P3, P5}, at a cost of 9, an LP bound of 8 without the crewbase constraints and an LP bound of 9 with them. These agree with the chapter's optimum and with Tables VII and IX.

VI. RESULTS AND INTERPRETATION

A. The Optimal Plan

The optimal plan uses two pairings. P3 starts and ends at D, with a crew flying D to A, A to B, and B to C to D over three days. P5 starts and ends at A, with a crew flying A to B, B to C and C to A. Together they are paid 9 hours for 7 hours of flying. The flying-time lower bound is 7 hours, so pay-and-credit is 2 hours, or 28.6 per cent of flying time. All of it comes from P5, which is paid 5 hours for 3 hours of flying. This is far above the 1 to 2 per cent the chapter reports for real problems. The example has only seven flights and the costs are chosen by the chapter, so the figure shows the idea of pay-and-credit and not real performance.

B. Bounds and the Optimal Solution

Measure	Value (hours)	Above flying time
Flying-time lower bound	7	0.0%
LP relaxation without crewbase constraints	8	14.3%
LP relaxation with crewbase constraints	9	28.6%
Optimal integer solution	9	28.6%

Table XIII. Bounds and the optimal solution for the chapter's example.

Without the crewbase constraints the LP bound is 1 hour below the integer optimum, which is 11.1 per cent of 9. This integrality gap is the reason a branching rule is needed, even though the crewbase constraints remove it in this example.

C. Results Reported in the Chapter

Topic	Result	Source
Quality of pairing solutions	From 10 to 15 per cent above the flying-time bound to at most 1 to 2 per cent; about \$50 million a year for a large airline	Barnhart et al. (2003)
Planned and operated cost	Pay-and-credit rises from below 1 to about 4 per cent (large fleets) and from about 3 to about 8 per cent (smaller fleets)	Barnhart et al. (2003)
Flight retiming	Moving departures by at most 5 minutes cuts pay-and-credit by 25 per cent on average, and by at most 10 minutes by 35 per cent	Klabjan et al. (2002)
Regularity	Crew cost 10 to 40 per cent lower, and regularity 40 to 60 per cent higher	Klabjan et al. (2001a), as reported by Barnhart et al. (2003)
Move-up crews	Slightly higher crew cost but 5 to 10 times more move-up crews	Klabjan et al. (2001c); Chebalov and Klabjan (2002)
Maintenance and crew together	Crew cost of 7 if maintenance solution 1 is fixed first, and 5 with solution 2 or the integrated model	Cohn and Barnhart (2003)
Rostering run times	Ryan: 55 crew members, 120 pairings, up to 300,000 variables, under 10 minutes to 3 hours. Gamache et al.: up to 108 pilots and 568 pairings, 1 to 8 hours	Ryan (1992); Gamache et al. (1998)
Pairing run times	10 to 20 hours of CPU time for 300 legs (hub-and-spoke) or 2,000 legs (point-to-point), even approximately	Barnhart et al. (2003)

Table XIV. Results quoted in the chapter.

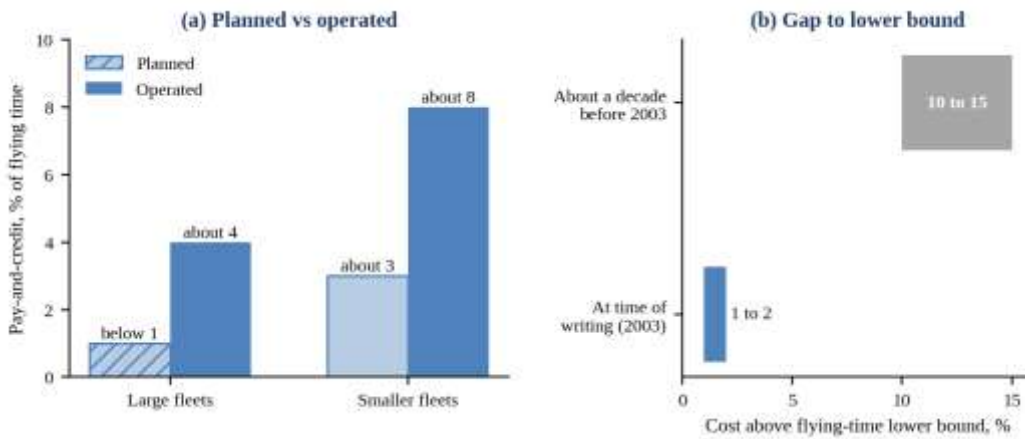


Figure 4. Cost gaps reported in the chapter: (a) planned and operated pay-and-credit; (b) gap to the flying-time bound. Source: Barnhart et al. (2003).

Three points follow. First, the percentages for retiming and regularity are relative to pay-and-credit or to crew cost in the studies concerned, so they should not be read as savings in total crew pay. Second, the gap between plan and operation is larger than the gap left to optimise. Moving from 10 to 15 per cent down to 1 to 2 per cent removes about 8 to 14 points of flying time, but operation then adds about 3 points for large fleets and 5 for smaller ones, which is more than the 1 to 2 points that remain. The room for improvement now lies as much in robustness as in better optimisation.

Third, the integrated example shows the cost of solving linked problems one after the other. In Figure 14.5 of the chapter, eight flights A to H have four candidate pairings costing \$1, \$2, \$5 and \$4. Each pairing needs particular forced turns, which are decided by the maintenance solution. Maintenance solution 1 allows only pairings 2 and 3, at \$7, and solution 2 allows pairings 1 and 4, at \$5. Fixing maintenance first can therefore cost 40 per cent more than solving the two together, and the integrated solution saves 28.6 per cent against the worse sequential outcome.

D. The Indian Context

Idea from the chapter	What the Indian case shows	Source
Work rules shrink the set of legal pairings (Section 14.2)	From 1 November 2025 weekly rest rose to 48 hours and night landings were limited to two, against six earlier, so rosters had to be rebuilt for tighter limits	PTI (2025); Storyboard18 (2025)
Plans with no buffer are fragile (Section 14.7)	The inquiry found rosters designed to maximise duty periods, with reliance on deadheading, tail swaps, extended duty and minimal recovery margins	PIB (2026)
Recovery must be fast (Section 14.6)	The DGCA allowed temporary relaxations, including up to six night landings and leave counted as weekly rest, until 10 February 2026. IndiGo restored operations to normal levels in a short time	Storyboard18 (2025); PIB (2026)
Failure is costly	A penalty of Rs 22.20 crore (Rs 1.80 crore one-time plus Rs 20.40 crore for 68 days at Rs 30 lakh a day) and a Rs 50 crore bank guarantee released in stages, of which Rs 15 crore is linked to manpower planning, rostering and fatigue-risk management	PIB (2026)
Rule changes force re-solving (Section 14.4.3.2)	A change in a feasibility rule can force major changes to shortest path code, which is why vendors prefer listing pairings. The inquiry also found deficiencies in system software support	Barnhart et al. (2003); PIB (2026)

Table XV. The chapter's ideas and the IndiGo case.

The link is interpretive. The published findings blame over-optimisation, inadequate preparation for the new rules, software deficiencies and management structure, and they do not point to an optimisation model as such. The pattern is nevertheless the one the chapter describes: a plan that spends every legal minute of duty has no slack when the rules tighten or flights run late.

E. Managerial Implications

- **Plan for operation, not only for the plan.** Pay-and-credit rises from below 1 to about 4 per cent for large fleets when flown, so a schedule with a slightly higher planned cost can be cheaper in operation. The chapter reports that move-up crews add robustness for a small rise in cost (Klabjan et al., 2001c).
- **Do not treat the legal limit as the target.** The Indian inquiry found rosters built to maximise duty periods with minimal recovery margins (PIB, 2026). Tight connections are the first to break, and the chapter reports that schedules are more robust when pay-and-credit has low variance and few pairings have zero pay-and-credit (Schaefer et al., 2000).
- **Solve linked problems together where possible.** Fixing maintenance first can cost 40 per cent more in the chapter's example, and retiming departures by 5 minutes cuts pay-and-credit by 25 per cent.
- **Keep the rules as inputs.** The model has to be solved again when regulations change, and pricing by listing pairings is easier to update than shortest path code (Barnhart et al., 2003).

VII. CRITICAL ANALYSIS

This part looks at the whole study. It sets out what was found and discusses the strengths and the weaknesses of the chapter and of this report.

A. Key Findings

The most important finding is that crew pairing, which looks like an impossible listing problem, can be solved well by combining a set partitioning model, column generation and branch-and-price. The chapter reports solutions within 1 to

2 per cent of the flying-time bound, up from 10 to 15 per cent, and savings of about \$50 million a year for a large airline. Our re-working shows the same structure on a small scale: only one of the 32 combinations of pairings is feasible, and its cost is 9 hours.

The second finding is that the LP relaxation does not always give an integer answer. Without the crewbase constraints the bound is 8 and the best integer plan is 9. The crewbase constraints close the gap here, but that is a feature of this example. In general branching is needed, and it has to be compatible with the pricing network, which is why follow-on and timeline branching were developed.

The third finding is that cost in the plan and cost in operation are different things. Pay-and-credit rises from below 1 to about 4 per cent for large fleets, and the Indian case shows the extreme: rosters that were legal on paper but had no buffer failed when the rules tightened and flights slipped. Sequential planning also loses money, since fixing maintenance first can cost 40 per cent more in the chapter's example.

B. Strengths

- **Coverage:** it handles the whole problem in one place, from building blocks and pay rules to formulations, algorithms, integration, recovery and robustness, with consistent notation.
- **Numbers with methods:** instance sizes, run times and percentage gains are quoted for most methods, which lets a reader judge what each method can do.
- **A worked example that can be checked:** it is small enough to solve by hand, and we could confirm every figure in it apart from the small slips noted below.
- **Honesty about open issues:** the chapter says that the best network type, the number of columns to add and the literature on recovery are unsettled (Sections 14.4.6 and 14.6.2).
- **Robustness as a modelling issue:** it evaluates schedules by simulation and adds move-up crews to the model, instead of leaving disruption to the operations room.

C. Limitations

- **Dated:** the chapter was written around 2002 and cites nothing later. Its run times of 10 to 20 hours belong to the computers of that time, and Kasirzadeh et al. (2017) later published data sets and new results.
- **United States and cockpit crews:** the pay rules, the 8-in-24 rule and the hub-and-spoke emphasis are American, and cabin crews are outside its scope. Indian rules such as the 48-hour weekly rest and the night landing limit are not covered.
- **Source base:** 20 of the 69 references have a chapter author, and 26 were technical reports, proceedings, presentations, a thesis or "to appear" items. Most results come from the authors' own work and cannot be checked from the chapter alone.
- **Results as ranges:** figures such as 10 to 15 per cent, 1 to 2 per cent and 10 to 40 per cent come without instance data, so they are hard to reproduce, and the chapter has no public benchmark.
- **An internal tension:** Section 14.8 says solutions are within at most 1 to 2 per cent of the bound, while Section 14.7 gives a planned pay-and-credit of about 3 per cent for smaller fleets. The 1 to 2 per cent is best read as applying to large-fleet daily problems.
- **Small slips:** (i) the text gives the label (4.0, 7.0, 4, 1), which should be (4.0, 6.0, 4, 1); (ii) the recovery model defines a variable v_p but uses v_k ; (iii) the crewbase limits of the example are called hours of pay, but the coefficients are flying hours (3 for P5, whose cost is 5); (iv) the printed pairing P6 does not form a connected chain of duties. None of them changes a conclusion.
- **Limits of this report:** we re-worked only the seven-flight example and did not implement column generation or the recovery model. The link to the IndiGo case is interpretive and rests on a regulator's press release and news reports. The duty pay illustration uses an assumed 5/8 fraction and minimum guarantee.

D. Possible Extensions

The most useful extension for an Indian airline is to put the DGCA's rules into the pricing step. The 48-hour weekly rest, the limit of two night landings and the limits on night duty would become label limits or enumeration rules, and the extra pay-and-credit caused by the stricter rules could be measured against the old ones.

A second extension is to test robustness by simulation using Indian delay data, as Schaefer et al. (2000) did with a simulation of airline operations, and to add a buffer rule that stops rosters being built at the legal limit. A third is to treat cabin crews, which the chapter leaves out, and to solve pairing and recovery together. Finally, the public data sets of Kasirzadeh et al. (2017) could be used to repeat the comparisons in Table XIV.

VIII. CONCLUSION

This report studied the chapter on airline crew scheduling by Barnhart et al. (2003) as an operations research case study. Crew pairing is modelled as a set partitioning problem in which every flight is covered by exactly one pairing at the least pay above flying time. Because the number of legal pairings runs into billions, the chapter solves it with column generation and branch-and-price, and it extends the models to rostering, recovery, integration with maintenance and schedule planning, and robustness.

We re-worked the chapter's seven-flight example. Only one of the 32 combinations of pairings, P3 with P5, covers every flight exactly once, at 9 hours of pay for 7 hours of flying. The LP bound is 8 without the crewbase constraints and 9 with them, the weekly repair leaves 11 flights to cover, and the label update in the shortest path example keeps two labels because neither dominates. These checks agree with the chapter apart from a small slip in one label.

Three lessons stand out for managers. First, cost is measured above flying time, and the plan that looks cheapest on paper is not the cheapest when flown, since pay-and-credit rises from below 1 to about 4 per cent for large fleets. Second, linked problems should be solved together, because the chapter's example gives 7 against 5 when maintenance is fixed first. Third, rules are inputs that change. India's 2025 duty rules and the IndiGo disruption, which cancelled 2,507 flights in three days, show the cost of rosters that run at the limit with no buffer.

The chapter has limits. It is dated, centred on the United States and on cockpit crews, relies heavily on its authors' own work and gives many results as ranges. This report shares the limits of working only with the chapter's small example and of linking the Indian case by interpretation.

Even so, the chapter shows how a first-course tool, set partitioning solved by linear programming and branching, can be scaled by column generation to a problem with billions of variables. With Indian rules as inputs and simulation to test robustness, the same approach could become a practical planning tool for Indian airlines.

APPENDIX: KEY TERMS

Term	Meaning
Crewbase	The home airport of a crew. Every pairing starts and ends at a crewbase.
Duty period	A series of flights flown by one crew in a working day, with minimum and maximum waits between flights.
Pairing	A trip of one or more duties separated by rest, starting and ending at the crewbase.
Schedule or roster	A sequence of pairings with time off for one crew member, usually for a month.
Deadhead	A crew member flying as a passenger to reach the place where the next duty starts.
Block time	The flying time counted against legal limits on a duty or a pairing.
TAFB	Time away from base, measured over the whole pairing.

Term	Meaning
Pay-and-credit	Pay above the actual flying time. It is the quantity that crew pairing tries to minimise.
Set partitioning	A model in which every row (flight) is covered by exactly one chosen column (pairing).
LP relaxation	The same model with the 0-1 requirement dropped. Its value is a lower bound on the integer optimum.
Column generation	Solving an LP with a huge number of columns by working with a few and adding more when they can lower the cost.
Pricing subproblem	The search for a column with negative reduced cost, usually a constrained shortest path.
Label and dominance	Label: a record of resources used on a path. A path is dominated if another is at least as good on every label.
Branch-and-price	Branch-and-bound in which columns are generated at every node of the tree.
Follow-on branching	A branching rule that forces two flights to be consecutive in a pairing, or forbids it.
Forced turn	A connection that is legal for a crew only because both flights use the same aircraft.
Move-up crew	A crew that could swap with a delayed crew, which gives a way to recover.
FDTL	Flight duty time limitations: the regulator's limits on flying, duty and rest. In India they are issued by the DGCA.

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