

Linear Programming Models for Investment Portfolio Allocation

Formulation, Diversification Constraints and a Re-examination of a Nigerian Stock-Selection Case

Submitted by:

Arnav Vaishnav, Aryaa Goenka, Avika Doneria, Bhavya Gandhi, Delisha Dhingra

Course mentor: **Prof. Tejaswini Angre**

Course: Operations Research

B.Sc. Finance Program, Anil Surendra Modi School of Commerce, NMIMS, Mumbai



<https://doi.org/10.55041/ijssmt.v2i10.005>

Cite this Article: Vaishnav, A., Goenka, A., Doneria, A., Gandhi, B. & Dhingra, D. (2026). Linear Programming Models for Investment Portfolio Allocation. *International Journal of Science, Strategic Management and Technology*, 02(10), 1-9. <https://doi.org/10.55041/ijssmt.v2i10.005>

License:  This article is published under the Creative Commons Attribution 4.0 International License (CC BY 4.0), permitting use, distribution, and reproduction in any medium, provided the original author(s) and source are properly credited.

Abstract

Investment portfolio allocation focuses on the allocation of a given capital investment among different assets in the presence of risk, return and liquidity. The literature review discusses the application of Linear Programming (LP) in the above decision making problem ranging from Markowitz's quadratic model to linear models such as mean absolute deviation (MAD). This includes the LP stock selection of Mbah and Onwukwe (2016) that evaluates five Nigerian stocks based on dividend, risk and liquidity. The published model is reviewed and solved using an LP solver. It reproduces the same choices for Zenith Bank, Fidelity Bank and Diamond Bank but the audit indicates that the reported "amounts invested" and optimal solutions are results of the normalisation in the model, that every LP allocates the entire capital to a single stock, and that the initial choice varies when liquidity is in ₦ billion instead of ₦ trillion. Two budget-constrained LPs are therefore built on the same data: one with a maximum weight per stock, and one that adds a risk ceiling and a liquidity floor. A 40% cap gives an allocation of 40-40-20 to the same three banks in a single LP. The addition of the liquidity floor moves about 7% of capital into the most liquid stock, which also carries the highest risk. The selected set does not change in all 10,000 random tests of up to $\pm 30\%$, but the ranking of stocks is highly dependent on the defining system of the score. A MAD-based LP could be a proposed extension but it would require return time-series data and has not been solved here. The outcome or results show that LP can be a transparent baseline. Its usefulness depends on constraints, it does not by itself prove investment performance.

Keywords: operations research; linear programming; portfolio allocation; diversification; liquidity; mean absolute deviation; sensitivity analysis

1. Introduction

Mathematical models are used in Operations Research (OR) to make decisions in which scarce resources must be shared among competing uses. Linear Programming is the tool used most widely within it- a decision is expressed as a linear objective subject to linear constraints and then solved to optimality. Portfolio allocation exhibits the same structure. An investor has a fixed fund and must determine how much to allocate to each asset, balancing return against risk, liquidity and policy limits.

Markowitz (1952) used variance to measure risk, which makes it a quadratic problem. Subsequent research demonstrated that the problem can be reformulated as an LP, which is simpler to solve, scale and explain. This has turned LP into a standard baseline in applied studies. A baseline is useful only when its behaviour is understood. A simple LP lacking any diversification rule picks a vertex of its feasible region and therefore tends to put the whole fund into a single asset. This study asks whether published LP applications protect against this, and how sensitive their conclusions turn out to be.

The study includes two related objectives. Firstly, it organizes some literature on LP portfolio allocation. Secondly, it resolves, reviews and expands the work of Mbah and Onwukwe (2016), which provides a clear numerical example with complete data. Each statement will be labeled according to the type of evidence that it represents: [S] is taken from the cited source, [C] is calculated in this paper and

[P] is proposed without being implemented. The study answers five research questions:

- RQ1. How does the formulation of the problem of portfolio allocation as LP look like and how has it changed in literature from quadratic to linear risk modeling?
- RQ2. What are the constraints needed for an LP not to give rise to a single-stock portfolio?
- RQ3. What are the findings of the Mbah and Onwukwe case once resolved and audited?
- RQ4. How do the diversification, risk and liquidity constraints affect the allocation process and what is the robustness of the result with respect to the data and the scoring function?
- RQ5. What are the limitations of such models and how can they be expanded while remaining linear?

2. Review Scope and Approach

This is a focused narrative review, not a systematic review or meta-analysis. Sources were chosen for direct relevance to LP-based allocation and fall into four groups: foundational theory (Markowitz, 1952; Sharpe, 1971; Konno and Yamazaki, 1991; Rockafellar and Uryasev, 2000), a review of twenty years of LP portfolio models (Mansini, Ogryczak and Speranza, 2014), applied LP studies (Oladejo, Abolarinwa and Salawu, 2020; Ogboi et al., 2025; Silva, Moreira and Francisco, 2014; Mbah and Onwukwe, 2016), and empirical evidence on estimation error (DeMiguel, Garlappi and Uppal, 2009). Mbah and Onwukwe (2016) is the core application paper because it reports its data, model and results in full. The other applied studies are used for comparison of modelling choices, not re-analysed.

3. Literature Review

Portfolio allocation as an optimisation problem

Portfolio allocation divides a budget among n assets so as to maximise expected return, or minimise risk for a required return. Markowitz (1952) showed that assets should be judged jointly, because holding assets that do not move together lowers overall risk. With variance as the risk measure the model is a quadratic program that needs a full covariance matrix. In its simplest LP form the decision variables are the amounts or shares x_j in each asset, the objective is a linear function $\sum r_j x_j$ of expected returns, the budget constraint is $\sum x_j \leq B$ (or $= 1$ for shares), and risk, liquidity and policy limits enter as further linear constraints with $x_j \geq 0$.

Linear approaches to risk

The literature offers several ways of keeping the problem linear. Sharpe (1971) approximated the quadratic problem by a piecewise linear one that can be solved by LP. Konno and Yamazaki (1991) replaced variance by the mean absolute deviation of portfolio return, which turns the problem into an LP while still using historical co-movement of assets. Rockafellar and Uryasev (2000) developed optimisation of Conditional Value-at-Risk (CVaR), a downside measure; for scenario-based problems the calculations often reduce to LP. Mansini, Ogryczak and Speranza (2014) review twenty years of such LP-solvable measures and note that adding realistic features such as transaction lots typically leads to mixed-integer models. Table 1 summarises the options relevant to this study.

Table 1. Linear and near-linear ways of treating risk in portfolio allocation [S]

Approach	How risk is handled	Remains an LP?	Data required
Mean–variance (Markowitz, 1952)	Variance of portfolio return	No, quadratic	Means and covariance matrix
Piecewise-linear approximation (Sharpe, 1971)	Linear approximation of the quadratic risk function	Yes (approximation)	As mean–variance
Average risk score or limit	Weighted-average score kept below a ceiling	Yes	One risk score per asset
MAD (Konno and Yamazaki, 1991)	Mean absolute deviation of return	Yes	Historical return series
CVaR (Rockafellar and Uryasev, 2000)	Average loss in the worst scenarios	Yes, with scenarios	Return scenarios

What applied LP studies do

Applied studies show how constraints are used in practice. Oladejo, Abolarinwa and Salawu (2020) allocate a firm's funds among seven investment options with a 20% limit on any single option, minimum shares for some groups and a cap on the average risk score, solved by the simplex method in MATLAB. Ogboi et al. (2025) allocate funds among government bonds, real estate trusts and stocks in Excel Solver, with minimum allocations and an average-risk limit. Silva, Moreira and Francisco (2014) build an LP portfolio from Brazilian listed companies and compare it with the Bovespa index over 2007–2012. Mbah and Onwukwe (2016) rank five Nigerian stocks on dividend, risk and liquidity using a sequence of LPs. Two patterns emerge. Risk is usually handled by an average score or limit, a linear proxy that ignores how assets move together. And a concentration limit appears only where the authors impose one explicitly, as Oladejo et al. do.

Estimation error and naive diversification

DeMiguel, Garlappi and Uppal (2009) evaluate 14 mean–variance-based models on seven datasets and find that none is consistently better out of sample than the naive 1/N rule on Sharpe ratio, certainty-equivalent return or turnover; estimation error offsets the gain from optimisation. Two lessons apply here. An optimised allocation should be compared with 1/N, and it should be tested for sensitivity to its inputs, because the optimiser will exploit input errors.

Synthesis, literature-to-model chain and research gap

Table 2 links each literature finding to a modelling decision in this study, so that the model is a consequence of the review.

Table 2. From literature finding to model and test

Literature finding	Modelling decision	Formulation	How it is evaluated
An LP optimum lies at a vertex and concentrates capital; applied studies impose caps (Oladejo et al., 2020) [S]	Cap on any single holding	$x_j \leq u$	Weights, number of holdings, concentration index [C]
Applied studies use an average risk score or limit (Oladejo et al.; Ogboi et al.; Mbah and Onwukwe) [S]	Portfolio-average risk ceiling	$\sum R_j x_j \leq \bar{R}$	Slack or binding; dual value [C]
Among the applied studies reviewed, only Mbah and Onwukwe model liquidity [S]	Portfolio-average liquidity floor	$\sum L_j x_j \geq \underline{L}$	Effect on weights; infeasibility limit [C]
Estimation error limits optimised portfolios (DeMiguel et al., 2009) [S]	Compare with 1/N; test input sensitivity	$u = 20\%$ gives 1/N; random perturbation	Rank stability [C]
MAD captures co-movement and stays linear (Konno and Yamazaki, 1991) [S]	Proposed risk-aware extension	Model D, Section 10	Requires return data [P]

Research gap. Within the sources reviewed, simple LP applications typically report a single optimum. Diversification is either imposed ad hoc or absent, risk is a linear score, and the sensitivity of the answer to its inputs and units is rarely examined. The Mbah and Onwukwe case is an instructive example: the LP is used as a stock-ranking device, with no budget constraint, no diversification rule and no robustness analysis. What is missing is a documented path from such a baseline to a diversified, liquidity-aware LP, with evidence of how each constraint alters the portfolio and how stable the result is. The study provides a path based on the available data (Sections 5-8) and outlines the MAD extension that would add co-movement (Section 10). The gap is framed to the sources reviewed and is not an assertion about the entire literature.

4. Real-Life Decision Context

The decision-maker is an investor, individual or fund manager, in a developing equity market who has a fixed fund and a short list of candidate stocks. In such markets many firms operate on borrowed funds, so high profit does not always mean high dividends, and the ease of converting a holding to cash matters (Mbah and Onwukwe, 2016) [S]. Table 3 states the decision in OR terms.

Table 3. The allocation decision framed as an OR problem

Element	Specification in this study	OR counterpart
Decision	Share of capital placed in each of five stocks	Decision variables x_j
Objective	Maximise a composite attractiveness score (Models A–C); minimise MAD of return (Model D, proposed)	Objective function
Restrictions	Full investment, no short selling, maximum weight per stock, risk ceiling, liquidity floor	Linear constraints
Horizon and information	Single period; dividend, risk and liquidity treated as known	Deterministic model

5. Mathematical Models

Notation

Table 4. Variables and parameters

Symbol	Meaning	Unit
j	Index of stock: 1 Zenith Bank, 2 Fidelity Bank, 3 Diamond Bank, 4 Dangote Cement, 5 7up Bottling	–
x_j	Decision variable. Model A: unscaled amount; Models B–D: fraction of capital in stock j	Fraction (B–D)
D_j, R_j, L_j	Dividend per share, risk factor, liquidity of stock j [S]	₦, %, ₦ trillion
s_j	Composite score $D_j L_j / R_j$ (Mbah and Onwukwe, 2016) [S]	Mixed (index)
T_D, T_R, T_L	Totals of D, R, L over the stocks still in the problem	As factor
$u; \bar{R}; \underline{L}$	Maximum weight per stock; risk ceiling; liquidity floor	Fraction; %; ₦ trillion
r_{jt}, μ_j, y_t, ρ	Model D: return of stock j in period t , its mean, deviation variable, required mean return	Return

Model A: the published model

Mbah and Onwukwe (2016) give each stock the score $s_j = D_j L_j / R_j$, so that dividend and liquidity raise and risk lowers attractiveness. Each constraint coefficient is the stock's share of the factor total (for example $29.00/45.60 = 0.64$ for Zenith's dividend) and each right-hand side is the factor total [S]:

Maximise $Z = \sum_j s_j x_j$

subject to $\sum_j (D_j / T_D) x_j \leq T_D, \sum_j (R_j / T_R) x_j \leq T_R, \sum_j (L_j / T_L) x_j \leq T_L, x_j \geq 0$

The LP is solved, the selected stock is removed, totals and shares are recomputed, and the LP is solved again, three times in all.

Classification: continuous, pure, deterministic, single-period maximisation LP with five (then four, then three) variables, three constraints and no budget constraint. The sequence of three LPs is a screening device; it does not come from one optimisation over all stocks.

Model B: budget and concentration cap (proposed)

Maximise $Z = \sum_j s_j x_j$ subject to $\sum_j x_j = 1, 0 \leq x_j \leq u$ for all j

Here x_j is a genuine share of capital, so the output can be read as a portfolio. The cap u answers the concentration problem identified in Section 3: with $u = 100\%$ the model reproduces a single-stock corner solution and with $u = 1/n$ it forces equal weights. **Classification:** bounded, continuous, pure LP; deterministic; single-period. The score s_j is interpreted as a composite attractiveness index, not as an expected return.

Model C: risk ceiling and liquidity floor (proposed)

Model B constraints, plus $\sum_j R_j x_j \leq \bar{R}$ and $\sum_j L_j x_j \geq \underline{L}$

The ceiling and floor are set to the values of the equal-weight portfolio, $\bar{R} = 6.784\%$ and $\underline{L} = \text{₦}1.518$ trillion, so that the portfolio is required to be no riskier and no less liquid than the naive $1/N$ benchmark. This choice is a judgement, not a result from the literature. The weighted-average risk constraint is a linear proxy that ignores correlation.

Classification: bounded continuous pure LP, as Model B.

Model D: MAD-based allocation (proposed extension)

Model D follows Konno and Yamazaki (1991). With T historical return observations r_{jt} and sample means μ_j :

Minimise $(1/T) \sum_t y_t$

subject to $y_t \geq \sum_j (r_{jt} - \mu_j) x_j$, $y_t \geq -\sum_j (r_{jt} - \mu_j) x_j$ ($t = 1, \dots, T$),

$\sum_j \mu_j x_j \geq \rho$, $\sum_j x_j = 1$, $0 \leq x_j \leq u$, $\sum_{j \in S_k} x_j \leq v_k$ (optional sector limits)

Classification: pure continuous LP with $n + T$ variables and $2T + 2 + (\text{sector})$ constraints; deterministic (historical scenarios); single-period. It is not solved in this study because it requires a return time series, which the Mbah and Onwukwe data do not contain.

Assumptions

Table 5. Assumptions, limits and likely effects

Assumption	Why it is needed	Why it may fail	Likely effect
Parameters are known and fixed	LP needs numerical coefficients	Dividends, risk and liquidity change yearly; data year is not stated [S]	Optimum may not hold in another period
Single period	Keeps the model static	Real investors rebalance	No dynamic effects captured
Continuous, divisible holdings	Keeps the model an LP	Shares trade in whole lots	Small rounding error; lots would need integer LP
No short selling	$x_j \geq 0$	Some investors can short	Feasible set is smaller
No frictions	Avoids extra terms	Transaction costs, taxes, price impact exist	Overstates net attractiveness of reshuffling
Linear aggregation of risk and liquidity	Weighted averages stay linear	Risk of a portfolio is not the average of asset risks	Diversification benefit is understated
Score is a valid index	Gives an objective	Dividend per share ignores price; formula is a convention	Ranking depends on the formula (Section 8.3)

6. Solution Methodology and Validation

The source solves Model A by hand with the simplex method, and reports that each round reaches the optimum after one iteration [S]. In this study all models are re-solved with the HiGHS solver through the linprog routine of SciPy 1.17.1 in Python [C]. The objective vector, constraint matrices and bounds are entered directly; the solver returns a status, the primal solution, the objective value and dual values (shadow prices). Fractional weights are allowed, which is what makes the models continuous LPs.

Three distinct questions are kept apart, because a solver output answers only the first two (Table 6)

Table 6. Feasibility, optimality and validation

Level	Question	Test used here
Feasibility	Does the portfolio satisfy every constraint?	Solver status; residual of every constraint checked within 10^{-9}
Optimality	Is it the best solution of the model?	Solver status 0; dual values and reduced costs: an excluded stock has a score below the budget dual (the marginal stock's score)
Validation	Does the model make financial sense and are results reliable?	Replication of published figures; unit-invariance test; comparison with 1/N; rank robustness under random perturbation and under alternative scores

7. Case Study: Nigerian Stock Selection

Data

The data are those of Mbah and Onwukwe (2016), taken by the authors from companies' annual reports and Central Bank of Nigeria records. The year of the data is not stated and no price series is given. Three of the five stocks are banks. Dividend per share is used in place of a return rate, so share prices play no role.

Table 7. Dataset [S] and derived quantities [C]

Stock	D_j (₦/share)	R_j (%)	L_j (₦ trn)	s_j	D_j/R_j
Zenith Bank	29.00	4.20	1.85	12.77	6.90
Fidelity Bank	4.06	1.19	1.23	4.20	3.41
Diamond Bank	4.34	1.67	1.12	2.91	2.60
Dangote Cement	6.00	20.83	2.00	0.58	0.29
7up Bottling	2.20	6.03	1.39	0.51	0.36
Total	45.60	33.92	7.59	—	—

Replication of Model A

Table 8 compares the published results with this study's re-solution. With coefficients rounded to two decimals, as in the paper, the replication matches the published amounts and values in rounds 2 and 3 and is close in round 1; with unrounded coefficients the values differ slightly. In all versions the same three stocks are chosen in the same order, and the liquidity constraint is the only binding one in every round [C].

Table 8. Model A: Published & Re-solved results (amount x / optimal value Z)

Round	Stock chosen	Published [S]	Re-solved, 2-dp coefficients [C]	Re-solved, unrounded [C]
1	Zenith Bank	31.65 / 404.17	31.62 / 403.85	31.14 / 397.77
2	Fidelity Bank	27.33 / 114.79	27.33 / 114.80	26.79 / 112.41
3	Diamond Bank	18.04 / 52.50	18.04 / 52.50	18.16 / 52.86

Audit of the Published Model

Auditing [C] identifies three characteristics of Model A which are not addressed by the source.

(i) The amounts are scaling artifacts. Since each coefficient equals v_{jk}/T_k and each right hand side equals T_k , the constraint implies that $x_j \leq T_k^2/v_{jk}$ if stock j is held alone. This means that the reported amounts are actually these bounds, and not amounts of money or weights. Table 9 demonstrates this for Round 1: for Zenith, $7.59^2/1.85 = 31.14$. The optimal amounts cannot be compared from one round to another since there are different totals.

Table 9. Round 1: implied bound on holding each stock alone & resulting Z [C]

Stock	s_j	Bound: dividend	Bound: risk	Bound: liquidity	$Z = s_j \times \text{smallest bound}$
Zenith Bank	12.77	71.7	273.9	31.14	397.8
Fidelity Bank	4.20	512.2	966.9	46.84	196.6
Diamond Bank	2.91	479.1	689.0	51.44	149.7
Dangote Cement	0.58	346.6	55.2	28.80	16.6
7up Bottling	0.51	945.2	190.8	41.44	21.0

(ii) **Liquidity is cancelled out of the ranking.** Liquidity is the tightest restriction for each stock and therefore restricts. In this case, Z_j

$= s_j \times TL2/L_j = TL2 D_j/R_j$: The term L_j is in the formula and in the binding restriction and cancels. Using the published units, the selection of the three stocks is equivalent to ranking by the dividend/risk ratio, even though the model employs three restrictions.

(iii) **The result depends on the unit of measurement.** Since the bounds depend on T^2 , changing a factor's unit changes which constraint binds. Re-solving with liquidity in ₦ billion instead of ₦ trillion, an equally valid expression of the same data, makes the dividend constraint bind and changes the selection sequence from Zenith, Fidelity, Diamond to Fidelity, Diamond, Zenith. A model whose first choice changes with the unit of a variable is not unit-invariant, so Model A's ranking cannot be interpreted as a pure finance result. Models B and C do not have this defect: rescaling one factor multiplies every s_j by the same constant, and the thresholds $\bar{R}$ and $\underline{L}$ are expressed in the same unit as the factor they limit.

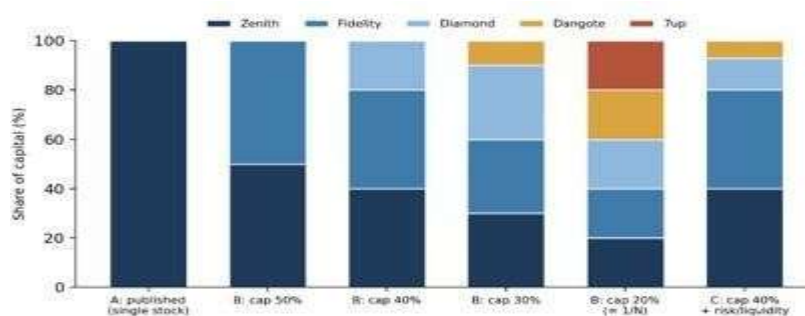
8. Results and Analysis

Model B: effect of the concentration cap

Table 10 and Figure 1 illustrate the optimal capital allocation as the cap u is narrowed [C]. With $u = 100\%$ the model yields solely Zenith Bank which conforms to the first published round behaviour. As u decreases the capital is spread over the score rank: for each LP its best-scoring stocks are filled to the cap. At $u = 40\%$ the allocation is 40% Zenith, 40% Fidelity, 20% Diamond, the same three banks, in the same order, that Model A needs three separate LPs to identify, but now as a single optimisation with weights that sum to one. Dangote Cement enters only when the cap falls below one third, and 7up Bottling only below 25%.

Table 10. Model B optimal allocations by cap [C]

Cap u	Zenith	Fidelity	Diamond	Dangote	7up	Score Z	Concentration (HHI)
100%	100%	0	0	0	0	12.77	1.000
50%	50%	50%	0	0	0	8.49	0.500
40%	40%	40%	20%	0	0	7.37	0.360
30%	30%	30%	30%	10%	0	6.02	0.280
25%	25%	25%	25%	25%	0	5.11	0.250
20% (1/N)	20%	20%	20%	20%	20%	4.19	0.200

Figure 1. Allocation under the published single-stock model (A), cap-constrained models (B) and the model with risk and liquidity limits (C) [C].


Diversification has a price in the model's own terms. Going from no cap to a 40% cap lowers the weighted score from 12.77 to 7.37, a fall of 42.3% (Figure 2). The dual values at $u = 40\%$ explain the trade-off: the budget constraint has dual 2.91, equal to Diamond Bank's score, because Diamond is the marginal stock; relaxing Zenith's cap by one unit of weight would add 9.86 to the objective, and Fidelity's 1.29. The score is an index, not a return, so this loss must not be read as a loss of investment return.

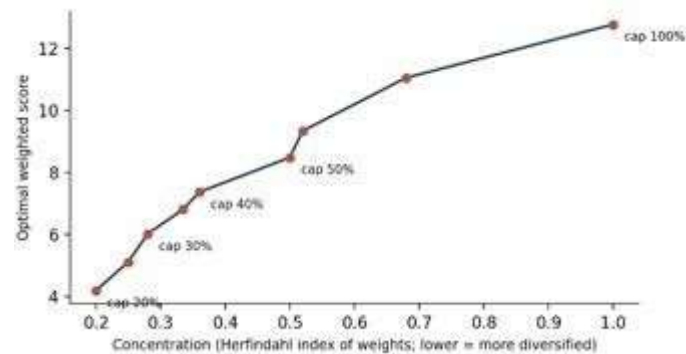


Figure 2. Trade-off between concentration and optimal score as the cap varies [C].

Model C: risk ceiling and liquidity floor

Table 11 compares four portfolios on their weighted characteristics [C]. Model B at $u = 40\%$ is well diversified and low-risk (average risk 2.49%), but its average liquidity, ₦1.456 trillion, is below that of the equal-weight portfolio (₦1.518 trillion). Imposing the liquidity floor in Model C changes the allocation to 40/40/13.0/7.1 across Zenith, Fidelity, Diamond and Dangote. Floor acts as the limiting factor; the risk ceiling is slack (average risk 3.84% versus a ceiling of 6.78%). The company that is the most liquid and the riskiest of all, Dangote Cement, is included in the portfolio for the sole purpose of satisfying the liquidity criterion. This comes at a minimal cost, as the score is reduced by just 2.2% from 7.37 to 7.21, and the dual value of the floor is 2.65 score units per ₦ trillion of average liquidity.

Table 11. Portfolio characteristics (weighted averages) [C]

Portfolio	Avg dividend (₦)	Avg risk (%)	Avg liquidity (₦ trn)	Score	Concentration (HHI)	Holdings
Equal weight (1/N)	9.12	6.78	1.518	4.19	0.200	5
A: published, round 1	29.00	4.20	1.850	12.77	1.000	1
B: cap 40%	14.09	2.49	1.456	7.37	0.360	3
C: cap 40%, risk and liquidity limits	14.21	3.84	1.518	7.21	0.342	4

Table 12 uses a different cap and liquidity floor with a risk ceiling of 6.784%. A liquidity floor of ₦1.4 trillion is non-binding for any value of cap that we tried; a liquidity floor of ₦1.518 trillion when $u = 50\%$ is also non-binding because even without it, Zenith and Fidelity would still satisfy the requirement. Increasing floors bring Dangote Cement into the portfolio, but for any liquidity above the maximum average, the problem becomes infeasible, and no portfolio can have an amount greater than ₦1.818 trillion when $u = 40\%$. Infeasibility is itself informative, since it shows when the investor's requirements are mutually inconsistent.

Table 12. Scenario analysis: cap u and liquidity floor L (₦ trillion) [C]

u	L	Score	Zenith	Fidelity	Diamond	Dangote	Status
30%	1.400	6.02	30%	30%	30%	10%	Optimal
30%	1.518	5.87	30%	30%	23.4%	16.6%	Optimal
30%	1.600	—	—	—	—	—	Infeasible
40%	1.400	7.37	40%	40%	20%	0	Optimal

40%	1.518	7.21	40%	40%	13.0%	7.1%	Optimal
40%	1.600	6.99	40%	40%	3.6%	16.4%	Optimal
40%	1.700	—	—	—	—	—	Infeasible
50%	1.518	8.49	50%	50%	0	0	Optimal
50%	1.700	7.73	50%	29.2%	0	20.8%	Optimal

Robustness of the ranking

Because the weights in Models B and C follow the ranking of s_j , the stability of that ranking is central. Two tests were run [C]. First, each of D_j , R_j and L_j for every stock was multiplied by an independent uniform random factor within $\pm 10\%$, $\pm 20\%$ and $\pm 30\%$, with 10,000 draws each (seed 2026). Zenith Bank ranked first in 100%, 100% and 99.7% of draws. The same three banks dominated in all levels of draws, but their order was preserved in only 99.7%, 89.5% and 79.6% draws respectively; Fidelity and Diamond changed places from time to time. Secondly, the score formula was diversified (Table 13).

Table 13. Ranking under Alternative Scoring Rules [C]

Score	Ranking (best to worst)	Same top three as base?
Base: $D \cdot L/R$	Zenith, Fidelity, Diamond, Dangote, 7up	—
D/R (no liquidity)	Zenith, Fidelity, Diamond, 7up, Dangote	Yes
$D \cdot L^2/R$ (liquidity weighted more)	Zenith, Fidelity, Diamond, Dangote, 7up	Yes
$D \cdot L/R^2$ (risk weighted more)	Fidelity, Zenith, Diamond, 7up, Dangote	Yes (order changes)
L/R	Fidelity, Diamond, Zenith, 7up, Dangote	Yes (order changes)
$D \cdot L$ (no risk)	Zenith, Dangote, Fidelity, Diamond, 7up	No: Dangote enters
D only	Zenith, Dangote, Diamond, Fidelity, 7up	No: Dangote enters

The choice of the three banks is robust to noise in the data, but not to the structure of the score. Dangote Cement only makes it into the top three if the risk is removed from the score, and the first ranked stock changes if risk is weighted more heavily. Data error is thus a smaller threat to the conclusion than the choice of scoring rule, which the source adopts without justification.

9. Critical Discussion

Strengths

The LP formulations are transparent because all factors, limitations, and rationale of the outcomes are explicitly stated. They are also easy to solve, and the dual values indicate the cost of constraints, which the ranking method cannot provide. The results matched the literature's essential methodological conclusion that diversification should be imposed by constraints, as noted in the study by Oladejo et al. (2020). In particular, Model B demonstrated it because the same input data resulted in a single stock without limits and a portfolio containing three to five stocks with constraints. Finally, the inclusion of liquidity as a factor is an innovation, as the majority of the cited works only consider Mbah and Onwukwe (2016).

Limitations

Table 14. Limitations tied to evidence from this study

Limitation	Evidence in this study	Consequence
Score is not a return	s_j is an index; dividend per share ignores price	Objective values cannot be read as investment performance
Deterministic, undated data	Year not stated; one cross-section	Allocation may not hold in another period
Linear risk proxy	Average risk ignores correlation; ceiling slack in Model C	Diversification benefit is not measured; Model D is needed
Judgemental thresholds	$\bar{R}$ and $\underline{L}$ set to equal-weight values	Results shift with thresholds (Table 12)
Small, concentrated universe	Five stocks, three banks	Sector limits cannot be meaningfully tested
No out-of-sample test	No return series	DeMiguel et al. (2009) warns that in-sample optima can disappoint
Source inconsistencies [S]	Round 3 constraints written with P instead of X; third table labelled “second best”; 31.64 vs 31.65; Z used for both objective and 7up	None changes the stocks selected; values from the simplex tables are used

A further limitation concerns Model A itself as described above: its measures are not weights, its cross round values are not comparable, and its ranking depends on units. These are properties of published formulation, not of LP, and are the main reason this study advocates budgeted and bounded formulations.

10. Proposed Extension and Future Research

The most defensible step forward is Model D, the MAD-based LP of Section 5.5, which would replace the average-risk proxy by a history-built co-movement measure while retaining a pure LP; this would require some computational implementation before numerical results could be reported, and here no results are claimed for it. It would be done as follows: [P] (1) collect adjusted closing prices of a well-documented set of stocks in various sectors within a certain time window; (2) calculate periodic returns and sample averages; (3) solve Model D for a grid of target return ρ to plot the trade-off between MAD and returns; (4) incorporate constraints of the sector and liquidity thresholds; (5) evaluate the performance of the portfolios against 1/N and Model B on a rolling out-of-sample window according to the framework of Silva et al. (2014) and with the reservations of DeMiguel et al. (2009); (6) report the solver status and constraints and dual values as outlined in Section 6. Further extensions which would break linearity or require integer variables, namely transaction costs with minimum lots, CVaR scenarios and multi-period rebalancing, are left for future work; integer lots would turn the model into a mixed-integer LP (Mansini et al., 2014).

11. Conclusion

RQ1. Portfolio allocation is naturally an LP in shares of capital, a budget constraint and linear limits; the literature moved from Markowitz’s quadratic model to linear risk treatments such as piecewise approximation, MAD and CVaR. RQ2. A concentration cap, together with a budget constraint, is the minimal device to prevent a single-asset corner solution; in the case data a 40% cap yields three holdings and a 20% cap enforces 1/N. RQ3. The Mbah and Onwukwe results are reproducible, but their amounts and values are a scaling artefact, liquidity cancels from the published ranking and the first choice varies with the unit of liquidity. RQ4. Adding in a liquidity floor reallocates about 7% of capital to the most liquid, highest-risk stock at a cost of 2.2% of the score; the choice of the three banks is stable under data noise of up to $\pm 30\%$ but sensitive to the definition of the score. RQ5. The main limits are deterministic data, a linear risk proxy and the absence of out-of-sample testing; MAD-based LP is the natural linear extension.

The results indicate that LP provides a transparent and computationally tractable framework for the modelled allocation problem, although its usefulness depends on the constraints imposed and the care taken over scaling and scoring; they make no claims about investment performance.

References

- DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *The Review of Financial Studies*, 22(5), 1915–1953. <https://doi.org/10.1093/rfs/hhm075>
- Konno, H., & Yamazaki, H. (1991). Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market. *Management Science*, 37(5), 519–531. <https://doi.org/10.1287/mnsc.37.5.519>
- Mansini, R., Ogryczak, W., & Speranza, M. G. (2014). Twenty years of linear programming based portfolio optimization. *European Journal of Operational Research*, 234(2), 518–535. <https://doi.org/10.1016/j.ejor.2013.08.035>
- Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1), 77–91. <https://doi.org/10.1111/j.1540-6261.1952.tb01525.x>
- Mbah, G. C. E., & Onwukwe, L. C. (2016). Linear programming method for portfolio selection and optimal financial investment in a developing economy. *International Journal of Applied Research*, 2(12), 338–342.
- Ogboi, C., Olurotimi, O., Olatunde, O. J., & Blessing, E. N. (2025). Application of linear programming model in investment portfolio and loan portfolio optimization. *International Journal of Economics, Business and Management Research*, 9(5), 447–463. <https://doi.org/10.51505/IJEBMR.2025.9529>
- Oladejo, N. K., Abolarinwa, A., & Salawu, S. O. (2020). Linear programming and its application techniques in optimizing portfolio selection of a firm. *Journal of Applied Mathematics*, 2020, Article 8817909. <https://doi.org/10.1155/2020/8817909>
- Rockafellar, R. T., & Uryasev, S. (2000). Optimization of conditional value-at-risk. *The Journal of Risk*, 2(3), 21–41.
- Sharpe, W. F. (1971). A linear programming approximation for the general portfolio analysis problem. *Journal of Financial and Quantitative Analysis*, 6(5), 1263–1275.
- Silva, P. M., Moreira, B. C. M., & Francisco, G. A. (2014). Linear programming applied to finance: Building a great portfolio investment. *Revista de Gestão, Finanças e Contabilidade*, 4(3), 107–124.