

Vaccine Distribution Optimisation using VAM and MODI — OR Literature Study

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1. Introduction

This study examines five published research papers on vaccine logistics and delivery in which Operations Research (OR) techniques are applied to real problems. The main topic is vaccine distribution optimisation: the least-cost movement of vaccine from sources to destinations, viewed through the transportation model and its two classic steps, an initial feasible solution (Vogel's Approximation Method, VAM) and an optimality test (MODI). The literature is then used to analyse, in detail, one published vaccine transshipment case and to show how its solution is checked for optimality. The focus is practical: vaccines are perishable, they need an unbroken cold chain, budgets are tight and volumes are huge. Guessing wastes doses; a model proves the best plan and shows the cost of every limit.

1.1 Operations Research

Operations Research turns a decision made under limits into a mathematical model, then solves it for the best possible answer. Picture a district officer with a fixed stock of vaccine vials, six health centres, one refrigerated van and a monthly budget. Every choice (who gets how many vials, by which route) has a cost and a limit. OR writes the choices as variables, the limits as constraints and the goal as an objective. Every OR model has the same three parts:

- **Decision variables:** what we control (vials shipped, who delivers which order).
- **Objective function:** what we minimise or maximise (cost, time, wasted doses).
- **Constraints:** what we cannot break (supply, demand, storage capacity, shelf life).

The linear programming (LP) template behind most of the models in this report is:

$$\min Z = \sum_j c_j x_j \quad \text{subject to} \quad \sum_i a_{ij} x_j \leq b_i \quad (i = 1, \dots, m), \quad x_j \geq 0$$

1.2 The transportation model: VAM and MODI

The transportation model ships goods from sources to destinations at least cost. It is solved in two stages. First an initial feasible solution is found (in this study, the role played by VAM). Then an optimality test (MODI) checks whether any unused route could lower the cost. The table below places the transportation approach among the problem types met in this report.

Problem type	Core question	Standard method	Paper
Transportation transshipment /	Ship goods from sources to destinations (via intermediaries) at least cost	Initial feasible solution + optimality test (MODI, Modified Exponential Approach)	Pasaribu et al. (2025)
Mixed integer linear programming (MILP)	Plan flows, stock, staff and trucks over time, with yes/no decisions	Branch and bound (CPLEX) with decomposition	Georgiadis and Georgiadis (2021)

Assignment	Match n agents to n tasks one-to-one at least cost	Hungarian algorithm	Vásconez et al. (2024)
Multi-criteria decision making (MCDM)	Rank competing criteria by importance	Modified Delphi + Fuzzy AHP	Lin et al. (2023)
Qualitative review	Which decisions matter in storage and distribution?	Literature synthesis	Scott et al. (2025)

VAM builds the starting plan from penalties: for each row and column the difference between its two smallest costs is the penalty for not using the cheapest cell, and the largest penalty is served first. MODI then assigns row and column values u_i and v_j to the occupied cells ($u_i + v_j = c_{ij}$) and tests every empty cell through its reduced cost $c_{ij} - u_i - v_j$. The plan is optimal when no reduced cost is negative. The central paper of this report uses a closely related penalty idea, the “exponent penalty”, and states that its allocation rule is consistent with the MODI optimality condition (Section 7).

1.3 Problem type and scope

Vaccine distribution can be approached from several perspectives: cost-minimising shipment (transportation), time-linked network planning (MILP), rider-to-order matching (assignment), and risk ranking (MCDM). This study adopts the transportation perspective as its core, because it is the setting in which an initial solution and a MODI optimality check apply directly. In the central case, a pharmaceutical distributor must ship a fixed monthly volume to six cities, either directly or through an intermediary. This is a transshipment problem, converted into a balanced transportation problem.

The original guideline called for 1–2 papers. The study deliberately expands the evidence base to five. Two are analysed in full depth (Pasaribu et al.; Georgiadis and Georgiadis). Three more are covered in shorter form to show the range of OR tools: an assignment model, a decision-analysis model and a context review.

2. Research Methodology

Five published studies on vaccine logistics and delivery were selected, from BAREKENG: Journal of Mathematics and Its Applications, Vaccine, Smart Cities, Mathematics and Outbreak Management and Response, published between 2021 and 2025. Four of the five apply a quantitative decision method to vaccine or delivery logistics; the fifth (Scott et al.) maps the real-world problems those methods must solve. Because the aim is to see how OR is applied to vaccine distribution, the papers were chosen to cover a range of tools rather than one technique.

Inclusion rules

- The paper applies an OR or decision-analysis technique, or reviews the problems such techniques must solve, for a real vaccine or delivery-logistics setting.
- It reports an allocation, plan, assignment, ranking or set of challenges that can be summarised and critiqued.
- Bibliographic details (journal, volume, pages, DOI) are available for the reference list.

Classification used for analysis

Theme	Link to vaccine distribution	Paper
A. Transportation / transshipment	Sets the least-cost routing of vaccine units from source to destination (central case)	Pasaribu et al. (2025)
B. Multi-period MILP network planning	Plans shipments, stock, shelf life, trucks and staff together over time	Georgiadis and Georgiadis (2021)
C. Assignment	Matches riders or vehicles to deliveries in the last mile	Vásconez et al. (2024)
D. MCDM / priority weighting	Ranks cold-chain transport risks so budgets go to the biggest ones first	Lin et al. (2023)
E. Qualitative review	Supplies the real-world problem the models solve	Scott et al. (2025)

For every paper, the following information was extracted: real-life problem, domain, objective, constraints, problem type, technique, software, data, validation and stated limitations. Several numerical inconsistencies in the source papers were checked against the papers' own tables and are reported in Section 8.

3. The Real-Life Problem

Delivering vaccine means moving a perishable product through a chain of dependent decisions before a dose reaches an arm. A poor plan costs money on every unit shipped, wastes doses, and can ruin vaccine in transit. The problem therefore belongs squarely within pharmaceutical distribution and public-health logistics.

3.1 What has to be decided

- **Routing and allocation:** who ships how many vials to which destination, directly or through an intermediary.
- **Storage and shelf life:** Pfizer needs deep freezers (-70°C), Moderna regular freezers (-20°C), and refrigerated Pfizer lasts only 5 days. Standard vaccines must stay at $2-8^{\circ}\text{C}$.
- **Last-mile delivery:** which rider or vehicle takes which order.
- **Risk priorities:** temperature excursions, handling errors and rough terrain can ruin vaccines in transit, so limited budgets must go to the biggest risks first.

Unreliable power, poor roads, thin monitoring and staff shortages drive wastage. Scott et al. cite a WHO finding that over 50% of vaccines are wasted globally, often through weak cold-chain management.

3.2 Why OR suits this problem

- Supply, demand and per-unit shipping costs are known, so the plan can be solved exactly. A cheaper route is a direct saving on every unit.
- There are many linked decisions (flows, stock, shelf life, staffing) with hard limits and yes/no choices, such as whether a truck runs today. Rules of thumb cannot balance these; an optimiser can.
- Last-mile delivery has one rider per order, a cost for each pairing and a goal of minimum total cost: the textbook assignment structure.
- Budgets are limited, and expert opinion is vague. Multi-criteria methods with fuzzy sets turn it into numerical priorities.
- Management needs specific decision outputs: the cheapest routing, the cost of every limit, and a plan that stays feasible when conditions change.

4. Summarised Analysis of the 5 Papers

Four of the five papers apply a quantitative method (a transshipment model, a MILP, an assignment model and a fuzzy MCDM method); the fifth is a narrative review with no model. Between them they use two software suites for optimisation (QM for Windows; GAMS with CPLEX), Python for data processing, and no software at all in the review.

#	Paper	Industry / setting	Real-life problem	OR technique	Software	Why OR fits
1	Pasaribu et al., BAREKENG 2025	Pharma distribution, West Kalimantan, Indonesia (PT IGM)	Ship 670,000 vaccine units to six cities, directly or via CV XYZ, at least cost	Transshipment to transportation model; Modified Exponential Approach	QM for Windows (check)	Fixed supply, known demand, per-unit costs: a textbook cost-minimisation LP
2	Georgiadis and Georgiadis,	National COVID-19 vaccine	Plan shipments, storage, trucks, staff and daily	MILP, two-step decomposition, rolling horizon	GAMS 30.1, CPLEX	Many interacting limits (cold storage, shelf life, targets) with

	Vaccine 2021	supply chain, Greece	appointments for four vaccines over 14 days			yes/no truck and route decisions
3	Vásconez et al., Smart Cities 2024	Last-mile delivery, Barcelona (MAX Delivery)	Assign delivery people to customer orders	Assignment problem; Hungarian algorithm; linear and polynomial regression for costs	Python	One-to-one matching with a cost matrix: the classic assignment structure
4	Lin et al., Mathematics 2023	COVID-19 vaccine cold-chain transport, China	Which risk factors matter most for transport quality?	Modified Delphi + Fuzzy AHP (MCDM)	Not stated	Expert judgement is vague; fuzzy pairwise weights rank criteria so budgets go to the biggest risks
5	Scott et al., Outbreak Mgmt & Response 2025	Vaccine storage and distribution in low-resource settings	Cold chain, last mile and wastage gaps	Narrative review (no model)	None	Identifies the decision problems OR can model

4.1 Theme A — Transportation and transshipment (Pasaribu et al.)

PT Indofarma Global Medika (PT IGM) must move 670,000 vaccine units in March to Pontianak, Ketapang, Singkawang, Sintang, Bengkayang and Kubu Raya. It can ship directly or route through an intermediary, CV XYZ. The paper finds the cheapest routing: PT IGM sends 209,000 units to CV XYZ, 151,000 to Pontianak and 310,000 to Kubu Raya, and CV XYZ forwards the rest to four cities. This is the central case of the study and is worked through in Section 6.

4.2 Theme B — Multi-period MILP planning (Georgiadis and Georgiadis)

The authors build one MILP that plans shipments, hub and clinic stock, truck fleet, staffing and daily appointments together over 14 days, and it keeps wasted doses low. Applied to a simulated Greek national campaign (5 hubs, 351 vaccination centres, 4 vaccines), storage cost is 78% of total cost and wasted doses only 0.7%. The model correctly sacrificed the cheapest vaccine (AstraZeneca) when doses had to be wasted, protecting costly mRNA vials.

4.3 Themes C, D and E — Assignment, MCDM and review

Vásconez et al. is not about vaccines, but it solves the matching step that every last-mile vaccine run also faces: which rider takes which order. Regression models estimate the missing distances and times, and the Hungarian algorithm then finds the cheapest one-to-one match. Using time as the cost, polynomial regression cut total delivery time by 17,143.41 min (11.59%) against linear regression. Lin et al. rank what matters most for safe cold-chain vaccine transport in China: fifteen experts' fuzzy judgements produce weights for 5 criteria and 26 sub-criteria, with environmental conditions carrying 36.2% of the weight and local weather alone the single most important sub-criterion (12.5%). Scott et al. supply the real-world problem that the models above solve: in low- and middle-income countries, broken cold chains and poor last-mile access waste vaccines.

4.4 Comparing the modelling approaches

The four quantitative papers sit on a ladder of complexity: a single-period cost model, a multi-period network model, a pure matching model and a ranking model.

Dimension	Pasaribu et al.	Georgiadis and Georgiadis	Vásconez et al.	Lin et al.
Objective	Minimise shipping cost	Minimise total chain cost incl. wasted doses	Minimise total distance or time	Rank criteria by weight (no cost objective)
Decision variables	Units x_{ij} per route	Flows, stock, vials used, staff, trucks, binary link choices	Binary x_{ij} match	Priority weights W
Key constraints	Supply, demand, balance	Stock balance, capacity, shelf life, targets, flow bounds, lead time	One rider per order, one order per rider	Consistency CR below 0.1
Problem class	Transshipment LP	MILP	Assignment (binary LP)	MCDM / fuzzy AHP
Solution method	Modified Exponential Approach	CPLEX, decomposition, rolling horizon	Hungarian algorithm + regression	Eigenvector method, fuzzy numbers
Tools	QM for Windows (check)	GAMS 30.1, CPLEX	Python	Not stated
Data	Company cost and demand data, March	Simulated Greek data (census, ministry of health list)	7,707 real orders	15 expert surveys
Validation	Same answer in QM; matches company practice	Optimality gap, cluster sensitivity, no-show test	Compared three cost matrices	Consistency ratio, alpha-beta sensitivity
Time dimension	Static	14-day rolling	Static batch	Static
Guarantees optimal?	Claimed, confirmed by software	Near-optimal (gap 5–8%) in the national case	Yes for the given matrix	Not applicable

Two patterns stand out. First, the more realistic the model, the more it leans on approximation: the national MILP accepts a 5–8% gap, while the small transportation model proves exact optimality. Second, models can be chained: Vásconez feeds machine-learning estimates into an exact optimiser, and Lin's weights could set priorities inside a cost model like Georgiadis's.

5. Modelling and Problem Formulation

The central problem is formulated as a transportation model: minimise total shipping cost while meeting every demand and respecting every supply. The other papers extend or complement this structure with time, matching and risk weighting.

5.1 Paper 1: transshipment as a balanced transportation problem

Let $i = 1$ be PT IGM and $i = 2$ be CV XYZ (both treated as sources). Let $j = 1$ be CV XYZ, and $j = 2 \dots 7$ the six cities (Pontianak, Ketapang, Singkawang, Sintang, Bengkayang, Kubu Raya). Define x_{ij} as units shipped from source i to destination j , c_{ij} as the unit cost (Rp), a_i as supply and b_j as demand.

$$\min Z = \sum_{i=1}^2 \sum_{j=1}^7 c_{ij} x_{ij}$$

$$\sum_j x_{ij} \leq a_i \quad (i = 1, 2), \quad \sum_i x_{ij} \geq b_j \quad (j = 1, \dots, 7), \quad x_{ij} \geq 0$$

Supply is $a_1 = a_2 = 670,000$. Demand is $b = (670,000; 151,000; 40,000; 55,000; 45,000; 69,000; 310,000)$, so total supply and total demand are both 1,340,000 and the table is balanced. The 670,000 “demand” at CV XYZ is a buffer: it lets goods pass through the intermediary, the standard trick for turning transshipment into transportation.

5.2 Paper 2: multi-period MILP

Indices and sets: f (plant), h (hub), vc (vaccination centre), v (vaccine: A, J, M, P), t (day, 1..14). **Key variables:** X (vials shipped between locations), S (vials in stock), VU (vials used), LS (vials lost), DA (appointments), HW and AH (health workers, base and additional), NT (trucks), and the binary $Y = 1$ if a shipment link is used on day t .

Objective (29): minimise total cost = storage (hubs and centres) + fuel + driver wages + wasted doses + extra healthcare staff + truck rental.

$$\min Z = C_{\text{storage}} + C_{\text{fuel}} + C_{\text{drivers}} + C_{\text{waste}} + r \sum AH(vc,t) + m \cdot NT$$

Main constraints (paper's numbering in brackets):

$$S(h,v,t) = S(h,v,t-1) + \sum f X(f,h,v,t) - \sum vc X(h,vc,v,t) - LS(h,v,t) \quad (4,5)$$

$$S(vc,v,t) = S(vc,v,t-1) + \sum h X(h,vc,v,t) - VU(vc,v,t) - LS(vc,v,t) \quad (6,7)$$

$$q_{\min}(i,j) \cdot Y(i,j,t) \leq \sum v X(i,j,v,t) \leq q_{\max}(i,j) \cdot Y(i,j,t) \quad (12,13)$$

$$\sum t DA(vc,t) = f(vc), \quad DA(vc,t) \leq g \cdot HW(vc,t), \quad AH(vc,t) \geq HW(vc,t) - i(vc) \quad (23-25)$$

$$WD(vc,v,t) = (VU^l(vc,v,t) - VU(vc,v,t)) \cdot d_v, \quad VU^l \text{ integer} \quad (27,28)$$

In words: hub and clinic stock balance each day; storage stays within technology capacity (10, 11); a safety stock remains at the end (9); shipments lie between minimum and maximum flows only if the link is active; every centre must complete its appointment target (23); a vaccination line of two workers does $g = 24$ jabs a day (6 h at 4 per hour), with extra lines costing more; and an open vial that is not fully used wastes its leftover doses. Shelf life (16–20) forces refrigerated Pfizer vials to be used within 5 days of arrival or counted as expired, and a one-day lead time (14, 15) delays use of vials shipped the day before.

5.3 Paper 3: assignment model

Let $x_{ij} = 1$ if delivery person i is assigned to order j , and c_{ij} the cost (distance in km or time in min).

$$\min Z = \sum_i \sum_j c_{ij} x_{ij} \quad \text{subject to} \quad \sum_j x_{ij} = 1 \quad \forall i, \quad \sum_i x_{ij} = 1 \quad \forall j, \quad x_{ij} \in \{0,1\}$$

Three ways were used to build the cost matrix c_{ij} : (1) Haversine distance from latitude and longitude ($R = 6,371$ km); (2) linear regression, $y = b_0 + b_1x_1 + \dots + b_nx_n$, fitted by gradient descent on mean squared error; and (3) polynomial regression of degree 2, $y = b_0 + b_1x + b_2x^2$, fitted the same way. The matrix used is 7,707 by 7,707.

5.4 Paper 4: fuzzy AHP weighting

An expert panel (6 logistics managers, 3 health professionals, 6 academics) scored indicators over three rounds of a Modified Delphi Method; literature gave 6 primary, 29 secondary and 173 tertiary indicators. An indicator was dropped if its mean score was below 4, kept if the mean exceeded 4 and the quartile deviation was below 0.6, and dropped if the quartile deviation exceeded 1.0. The final system has 5 criteria and 26 sub-criteria. Fifteen experts (5 industry, 4 government, 6 academia) then compared criteria pairwise on Saaty's 1–9 scale. For each pair, the answers become a triangular fuzzy number (L, M, U): the minimum, geometric mean and maximum of all expert scores.

$$L_{ij} = \min_k B_{ijk}, \quad M_{ij} = (\prod_k B_{ijk})^{(1/n)}, \quad U_{ij} = \max_k B_{ijk}$$

Each fuzzy number is defuzzified into a crisp value using an uncertainty level α and an optimism level β (both from 0 to 1):

$$g(\alpha, \beta) = \beta[(M - L)\alpha + L] + (1 - \beta)[U - (U - M)\alpha]$$

The weights are the principal eigenvector W of the pairwise matrix, with consistency checked by the ratio below (accepted if below 0.1). The overall weight of a sub-criterion is the criterion weight times the sub-criterion weight (for example S1: $0.362 \times 0.346 = 0.125$).

$$A \cdot W = \lambda_{\max} \cdot W, \quad CI = (\lambda_{\max} - n) / (n - 1), \quad CR = CI / RI$$

5.5 Paper 5: decisions hiding in the review

Scott et al. contain no mathematical model. The problems they describe, however, map onto the OR models above:

Problem in the review	OR model that fits	Seen in this report
Who ships how many vials by which route	Transportation transshipment LP	Pasaribu et al.
Stock, shelf life, storage technology, fleet and staff over time	Multi-period MILP	Georgiadis and Georgiadis
Matching vehicles or riders to deliveries	Assignment problem	Vásconez et al.
Which risk to fix first with a limited budget	MCDM (AHP)	Lin et al.
Where to place cold-chain points; drones versus trucks; sizing solar units	Facility location, routing and cost simulation	Not modelled here; the review cites such studies (Chen et al.; Lim et al.; Enayati et al.; Haidari et al.)

5.6 Assumptions

- **Paper 1:** costs are linear per unit; one month (March) is planned; no capacity limit at CV XYZ; no cold-chain or time constraints; costs stay fixed.
- **Paper 2:** one-way flow (plant to hub to centre); one truck visits one centre a day; no vehicle routing; homogeneous trucks; vial counts treated as continuous (rounded later); no vaccinations on Sundays; supply availability is not a bottleneck; wastage ratios follow WHO.
- **Paper 3:** straight-line distances stand in for road distance; traffic and speed are not modelled; each order needs exactly one rider.
- **Paper 4:** expert judgement is representative; criteria are independent; the base analysis uses $\alpha = \beta = 0.5$.

6. Worked Application: Vaccine Transshipment in West Kalimantan

The worked application takes the data of Pasaribu et al. (2025): PT IGM must supply 670,000 vaccine units in March, and the cheapest plan found costs Rp 13,536 million (Rp 13.536 billion). The allocation and unit costs below are those reported in the paper (the CV XYZ cost row is taken from the paper's Table 2, which reproduces the reported minimum cost). The MODI check in Section 6.2 is calculated from those figures.

6.1 Allocation and cost

Route	Units	Unit cost (Rp)	Cost (Rp million)
PT IGM to CV XYZ	209,000	5,000	1,045
PT IGM to Pontianak	151,000	6,000	906
PT IGM to Kubu Raya	310,000	12,000	3,720
CV XYZ to Ketapang	40,000	50,000	2,000
CV XYZ to Singkawang	55,000	30,000	1,650
CV XYZ to Sintang	45,000	40,000	1,800
CV XYZ to Bengkayang	69,000	35,000	2,415
Total	879,000		13,536

The 670,000 CV XYZ buffer self-flow (461,000 units, cost 0) is a modelling device, not a physical shipment. With it, PT IGM's 670,000 units (209,000 + 151,000 + 310,000) and CV XYZ's 670,000 units (461,000 + 40,000 + 55,000 + 45,000 + 69,000) both balance exactly.

6.2 MODI optimality check on the reported plan

The plan occupies 8 cells, equal to $m + n - 1 = 2 + 7 - 1$, so it is a non-degenerate basic solution and the MODI values can be found uniquely. Setting $u_2 = 0$ for CV XYZ and using $u_i + v_j = c_{ij}$ on every occupied cell:

Occupied cell	Cost c_{ij} (Rp)	Equation	Result
CV XYZ → CV XYZ (buffer)	0	$u_2 + v_1 = 0$	$v_1 = 0$
CV XYZ → Ketapang	50,000	$0 + v_3 = 50,000$	$v_3 = 50,000$
CV XYZ → Singkawang	30,000	$0 + v_4 = 30,000$	$v_4 = 30,000$
CV XYZ → Sintang	40,000	$0 + v_5 = 40,000$	$v_5 = 40,000$
CV XYZ → Bengkayang	35,000	$0 + v_6 = 35,000$	$v_6 = 35,000$
PT IGM → CV XYZ	5,000	$u_1 + 0 = 5,000$	$u_1 = 5,000$
PT IGM → Pontianak	6,000	$5,000 + v_2 = 6,000$	$v_2 = 1,000$
PT IGM → Kubu Raya	12,000	$5,000 + v_7 = 12,000$	$v_7 = 7,000$

Each empty cell is then tested by its reduced cost $c_{ij} - u_i - v_j$, which must be zero or positive for an optimal plan:

- **PT IGM → Ketapang, Singkawang, Sintang, Bengkayang:** the reduced cost is the direct cost minus 55,000, 35,000, 45,000 and 40,000 respectively, which are exactly the costs of going via CV XYZ (5,000 plus the CV XYZ leg). The paper states that routing via CV XYZ is cheaper in every case, which means each direct cost exceeds these figures and each reduced cost is positive.
- **CV XYZ → Pontianak and Kubu Raya:** the reduced costs are the CV XYZ cost minus 1,000 and 7,000. The paper's cost entries for these two cells are not reproduced in the source report, so these two tests remain open; PT IGM serves both cities directly, in line with its current practice.

The MODI check therefore supports the paper's optimal plan on the cells that can be tested from the available data. This is the dual ($u-v$) test that the paper itself does not show (Section 8.4).

6.3 Validation of the result

- QM for Windows returned the same allocation.
- The plan matches PT IGM's current practice: own vehicles to Pontianak and Kubu Raya, CV XYZ for the other four cities.
- The reported minimum cost of Rp 13,536,000,000 is reproduced by the Table 2 costs (the table above sums to Rp 13,536 million over 879,000 shipped units).

7. Solution Methodology Across the Papers

The reviewed studies solve their problems through a progression from a direct allocation method for a small transportation table to exact optimisation, decomposition, matching algorithms and expert-judgement weighting. A recurring feature is validation: authors compare their results with software output, optimality gaps or consistency checks.

7.1 Modified Exponential Approach (Paper 1)

A direct method that finds an optimal solution without the usual MODI loop. The authors refine an earlier version with two changes: (a) first check that every row and column holds a zero, and (b) when allocating, pick the cell with the smallest reduced cost (consistent with the MODI optimality condition). The 13 steps fall into three stages:

- **Initial feasible solution (steps 1–5):** balance the table, subtract the row minimum from each row, then the column minimum from each column, so every row and column has a zero.

- **Table revision (steps 6–8):** check each demand is not larger than the supply it can draw on; if not, cover zeros with minimum lines and adjust.
- **Optimality test (steps 9–12):** count zeros per row and column as an “exponent penalty”, allocate $x_{ij} = \min(a_i, b_j)$ at cells with penalty 1, then 2, then at the smallest reduced cost, and repeat until all supply and demand are zero.

In this case study the method needed three allocation rounds: 151,000 to Pontianak and 310,000 to Kubu Raya, then the rest of PT IGM's supply to CV XYZ, then CV XYZ's buffer and four city deliveries.

7.2 Other techniques

Technique	How it works	Papers
Modified Exponential Approach (transportation)	Row and column reductions, zero-count penalties and smallest-reduced-cost allocation until supply and demand are exhausted	Pasaribu et al.
Exact MILP (branch and bound)	CPLEX through GAMS 30.1 on small cases; the illustrative case (1 hub, 5 centres, 2 vaccines) reached its optimum of 22,059 RMUs in 30 CPU seconds	Georgiadis and Georgiadis
Two-step decomposition	(i) Give each centre to its nearest hub, group centres into clusters by political boundary, solve a small aggregated MILP per hub; (ii) fix all binary variables and solve the detailed LP for every centre	Georgiadis and Georgiadis
Rolling horizon	Prediction window 14 days (7 fixed, 7 flexible); re-plan after each day with actual appointments and stock	Georgiadis and Georgiadis
Hungarian algorithm + regression	Subtract row minima, then column minima, find a full set of independent zeros (adjusting with covering lines where needed); regression supplies the missing costs	Vásconez et al.
Modified Delphi + Fuzzy AHP	Delphi rounds filter indicators; fuzzy pairwise judgements are defuzzified and the principal eigenvector gives weights, checked by $CR < 0.1$	Lin et al.

7.3 Software and tools

- QM for Windows (Pasaribu et al.; marked “check” in the source report).
- GAMS 30.1 with the CPLEX solver, run on an Intel i7 with 16 GB of memory (Georgiadis and Georgiadis).
- Python for data processing (Vásconez et al.); the paper does not name the Hungarian implementation.
- Lin et al. and Scott et al. state no software.

7.4 How solutions were analysed and validated

- **Software cross-check.** QM for Windows returned the same allocation as the Modified Exponential Approach, and the plan matches company practice (Pasaribu et al.).
- **Optimality gap.** The national MILP used a 5% target; the aggregate step showed a 7–8% gap for three hubs and below 5% for two, and the detailed step below 1% (Georgiadis and Georgiadis).
- **Sensitivity analysis.** Georgiadis and Georgiadis varied the number of clusters; Lin et al. ran α and β from 0 to 1 to test how stable the ranking is.
- **Disturbance test.** Rolling-horizon re-planning was tested with up to 25% patient no-shows.
- **Comparison of alternatives.** Vásconez et al. compared total and average distance or time across three cost-matrix methods.
- **Consistency checking.** Lin et al. accepted pairwise matrices only with consistency ratio CR below 0.1.

8. Critical Analysis

OR gives vaccine logistics what experience alone cannot: a provably good plan and a price tag on every limit. All five papers add value, but each has gaps. Several are numerical inconsistencies that were checked against the papers' own tables.

8.1 Key findings

Paper	Result
Pasaribu et al.	Cheapest plan for 670,000 units costs Rp 13,536 million; PT IGM sends 209,000 units to CV XYZ, 151,000 to Pontianak and 310,000 to Kubu Raya; agrees with company practice
Georgiadis and Georgiadis — illustrative example	Optimum 22,059 RMUs in 30 s; storage 78.9% of cost, 26 doses lost (1.8%)
Georgiadis and Georgiadis — Greek study (351 centres, 54 clusters)	Aggregate-step gap 7–8% for three hubs, below 5% for two; detailed step below 1%; about 3.5 h in total, about 1 h in parallel
Georgiadis and Georgiadis — cost split	Storage 78%, drivers 9%, truck rental 8.4%, fuel 3.8%, wasted doses 0.7%, extra staff 0.3%
Georgiadis and Georgiadis — cluster sensitivity (hub H1)	12–15 clusters give feasible detailed plans; 10 or 11 clusters gave infeasible detailed solutions
Georgiadis and Georgiadis — rolling horizon (hub H1, 4 iterations)	Total cost 123,981; 112,517; 104,228; 104,359 despite up to 25% no-shows
Vásconez et al.	Polynomial regression + Hungarian cut total delivery time by 17,143.41 min (11.59%) against linear regression; Haversine + Hungarian gave the lowest distance (34,956.84 km)
Lin et al.	Environmental conditions rank first (0.362); local weather is the top sub-criterion (0.125)
Scott et al.	Cold chain, last mile and wastage gaps; solar refrigerators, sensors, drones and community health workers are the main remedies

Further detail on the last two papers:

Cost basis (Vásconez et al.)	Average	Std. dev.	Total
Hungarian on Haversine distance	4.55 km	3.24 km	34,956.84 km
Linear regression + Hungarian (distance)	4.55 km	3.23 km	34,957.74 km
Polynomial regression + Hungarian (distance)	4.62 km	4.49 km	35,526.26 km
Linear regression + Hungarian (time)	19.24 min	9.12 min	147,891.23 min
Polynomial regression + Hungarian (time)	17.01 min	5.25 min	130,747.82 min

Regression accuracy on its own favoured polynomial regression (time RMSE 7.7 s against 11.0 s). Yet for distance, linear regression gave the better final assignment. The lesson: the best predictor is not always the best input to an optimiser.

Criterion (Lin et al.)	Weight	Rank	Top sub-criterion (overall weight)
C1 Environmental conditions	0.362	1	S1 Local weather (0.125)
C2 Social and economic development	0.228	2	S7 Cold-chain standardisation (0.065)
C4 Safety-system management	0.173	3	S17 Local quality and safety systems (0.052)
C3 Infrastructure and equipment	0.156	4	S11 Road infrastructure (0.036)
C5 Quality and risk of transport	0.080	5	S22 Vaccine efficacy in transit (0.029)

Overall top five sub-criteria: local weather (S1), local terrain (S2), road features (S3), cold-chain standardisation (S7), national economic development (S6). As the future becomes more uncertain, safety monitoring, emergency response,

traffic status, specialist staff and information timeliness gain weight. Scott et al. report that vaccines must stay at 2–8 °C (standard) or as low as –70 °C (some mRNA vaccines); solar direct-drive refrigerators, phase-change materials, real-time sensors, drones (for example, the 2018 Vanuatu delivery) and community health workers are the main remedies. UNICEF supplied over 6,500 solar refrigerators between 2005 and 2012, and about 75% of 700 units in Sierra Leone still worked after eight years.

8.2 Impact and implications for vaccine distribution

- A simple transshipment model finds the cheapest route for 670,000 units and agrees with company practice, so exact methods can confirm and justify what a distributor already does.
- A MILP with decomposition can plan a whole national campaign, with storage (78% of cost) rather than transport or wastage as the main cost driver.
- Learned cost estimates can feed an exact assignment algorithm, though a better predictor does not always mean a better plan.
- Fuzzy expert judgement ranks transport risks when no cost data exist, so budgets can go to the biggest risks first.
- Model choice should follow the question: a transportation LP for cost, a MILP for time-linked planning, an assignment model for matching, MCDM for priorities.

8.3 Strengths of the approach

- **Pasaribu et al.:** clean, verifiable model; real company data; result matches current practice and an independent software run.
- **Georgiadis and Georgiadis:** first model to join vaccine distribution with appointment planning; models perishability and open-vial wastage explicitly; decomposition makes a national case solvable; rolling horizon handles no-shows.
- **Vásconez et al.:** real data (7,707 orders); sensible hybrid of machine learning and exact optimisation; clear comparison of three cost matrices.
- **Lin et al.:** structured consensus (MDM) before weighting; fuzzy numbers capture vague opinion; consistency checks; alpha–beta sensitivity analysis; transferable to other medicines.
- **Scott et al.:** broad, readable map of cold-chain, last-mile and community challenges.

8.4 Limitations and assumptions

- **Pasaribu et al.:** the abstract says Bengkayang receives 9,000 units, while the results and demand data say 69,000. The CV XYZ cost row in Table 1 (40,000; 55,000; 45,000; 69,000) differs from Table 2 (50,000; 30,000; 40,000; 35,000), and the Table 1 figures equal the demand figures, which suggests a copying slip. The reported Rp 13.536 billion reproduces only with the Table 2 costs; Table 1 costs give Rp 17.082 billion. The allocation is the same under both (via CV XYZ is cheaper in every case). Optimality is asserted from the algorithm and a software check; no dual (u–v) test is shown. The model is static and single-month, with no cold-chain, capacity or time limits, despite the paper's focus on vaccine quality.
- **Georgiadis and Georgiadis:** simulated data (real hub locations are secret); no vehicle routing and one centre per truck per day; vial counts treated as continuous; national solutions carry a 5–8% optimality gap; results depend on the number of clusters (10–11 clusters gave infeasible plans); no formal uncertainty model, only reactive re-planning; costs in relative units.
- **Vásconez et al.:** distance RMSEs do not match the stated MSEs (MSE 6.9 would give an RMSE near 2.6, not 20.3; MSE 2.6 gives about 1.6, not 4.5), and units mix metres and kilometres. The abstract's 1.51% saving for linear regression does not reproduce: 568.52 km is about 1.6% of the polynomial total. Plain Haversine plus Hungarian gives the lowest distance, so regression adds nothing for distance; the time results have no Haversine or real-world baseline, and nothing compares against the dispatch the company actually used. Straight-line distance ignores roads and traffic; the input variables of the regressions are not described; the 7,707 by 7,707 matrix implies as many riders as orders, which a real fleet does not have.
- **Lin et al.:** 15 experts and a China-only setting; weights are opinion, not measured risk; criteria are assumed independent. The text says the top five at $\alpha = \beta = 0.5$ stay stable (S22, S1, S6, S17, S11), but Table 10 ranks S1, S2,

S3, S7, S6 first, and S22 ranks 14th. The conclusion calls S23 important though it ranks 22nd of 26. “CR below 0.1%” is a slip for 0.1.

- **Scott et al.:** a narrative review with no search protocol, no model and no quantitative synthesis; some citations look unrelated to the claim they support (a zebrafish immunology paper behind Ebola logistics; a USDA meat-processing document for vaccine transport).

Reported figures need checking: several of the inconsistencies above would mislead a reader who copied the abstract.

8.5 Possibilities for extension

- **Pasaribu et al.:** add time windows and temperature-excursion risk, capacity at CV XYZ, and multiple months.
- **Georgiadis and Georgiadis (partly proposed by the authors):** routing with a metaheuristic such as Variable Neighbourhood Search, better clustering, stochastic or robust programming for no-shows and supply delays.
- **Vásconez et al.:** road-network distances (the authors suggest OSRM), dynamic and batched assignment, rider capacity and fairness, reinforcement learning.
- **Lin et al.:** fuzzy ANP for interdependent criteria; link the weights to a cost model so risk reduction is optimised under a budget.
- **Scott et al.:** a systematic review that pairs each challenge with a tested OR model.
- **Across the five papers:** none of them optimises cost, cold-chain risk and equity of access together. The natural next step is a stochastic, multi-objective MILP that uses Lin-style risk weights, Vásconez-style learned cost estimates and Georgiadis-style inventory and shelf-life constraints on real Indian or Indonesian district data.

9. Conclusion

OR gives vaccine logistics what experience alone cannot: a provably good plan and a price tag on every limit. Pasaribu et al. show that a simple transshipment model, solved as a balanced transportation problem and consistent with MODI optimality, finds the cheapest route for 670,000 units at Rp 13,536 million and agrees with company practice. Georgiadis and Georgiadis show that a MILP with decomposition can plan a whole national campaign, with storage (78% of cost) rather than transport or wastage as the main cost driver. Vásconez et al. show that learned cost estimates can feed an exact assignment algorithm, though a better predictor does not always mean a better plan. Lin et al. show how fuzzy expert judgement ranks transport risks when no cost data exist, and Scott et al. explain why these problems matter in low-resource settings.

The papers also teach caution. Model choice should follow the question: a transportation LP for cost, a MILP for time-linked planning, an assignment model for matching, MCDM for priorities. The best future work joins them, optimising cost, cold-chain risk and fairness of access together on real data.

10. References

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